

Calculation of the Critical Rocket Height for Upward Leader Inception in Rocket-Triggered Lightning

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Abstract: Determining the critical rocket height for upward-leader inception is important for evaluating the triggering conditions of rocket-triggered lightning and selecting appropriate experimental conditions. A two-dimensional axisymmetric electrostatic model of the rocket-triggering wire-ground system was established to investigate the critical rocket height for upward-leader inception in rocket-triggered lightning. The electric field at 0.7 m above the rocket tip was calculated under different rocket heights and cloud-to-ground background electric fields. Based on the upward-leader inception criterion, an electric field of 500 kV/m at this position was adopted as the threshold for determining the critical rocket height. The results showed that the electric field above the rocket tip increased approximately linearly with both rocket height and background electric field. A stronger background electric field significantly reduced the critical rocket height required for upward-leader inception. These results provide a quantitative basis for evaluating the triggering conditions of rocket-triggered lightning.

Keywords: Rocket-triggered lightning, Upward leader inception, Critical rocket height, Background electric field, Finite-element calculation.

1. Introduction

Artificially triggered lightning using rockets provides a controllable means of initiating lightning and has been widely used to investigate lightning discharge physics [1-3]. In the conventional rocket-triggered lightning technique, a rocket carries a grounded metallic wire into the thunderstorm electric field, causing the electric field near the rocket tip to increase with height [4,5]. When the local field becomes sufficiently strong, an upward leader is initiated from the rocket tip and develops toward the thundercloud, thereby triggering lightning [6-8]. For a given thunderstorm background electric field, the height at which the rocket tip first satisfies the condition for upward leader inception can be regarded as the critical rocket height. Determining this height under different background electric fields is important for understanding the initiation of rocket-triggered lightning and for selecting appropriate experimental conditions.

Previous studies on upward leader inception have mainly focused on field observations and discharge-physics models [9,10]. Willett et al. reported that a grounded triggering wire approximately 400 m long could trigger lightning when the ambient electric field aloft was about 13 kV/m [11]. Biagi et al. observed the inception of a sustained upward positive leader from a grounded triggering wire at a height of approximately 123 m [6]. These observations indicate that upward leader inception is closely related to both the thunderstorm electric field and the triggering-wire height. From the perspective of discharge physics, several physical criteria for upward leader inception have been proposed [12-15]. In the model of Petrov and D'Alessandro, positive streamers must extend to a critical length and experience an electric field sufficient to sustain their propagation. A critical streamer-zone length of approximately 0.7 m and a critical positive streamer field of approximately 500 kV/m have commonly been adopted [16-18]. However, quantitative relationships between the thunderstorm background electric

field and the corresponding critical rocket height remain insufficiently investigated.

In this study, a two-dimensional axisymmetric electrostatic model of the rocket-triggering wire-ground system was established. The local electric field at 0.7 m above the rocket tip is calculated for different combinations of rocket height and cloud-to-ground background electric field. A field strength of 500 kV/m at this position is used as the criterion for upward leader inception, and the corresponding critical rocket height is determined for different background electric fields. The results provide a quantitative basis for evaluating the triggering conditions of rocket-triggered lightning and selecting suitable experimental parameters.

2. Methods

2.1. Calculation of Background Electric Field near the Rocket Tip

Before upward-leader inception, the rocket-triggering wire-ground system was treated under an electrostatic approximation. Because the air domain was assumed to contain no free space charge, the electric potential V satisfied the Laplace equation,

$$\nabla^2 V = 0$$

And the electric field was obtained from

$$E = -\nabla V$$

The cloud-to-ground background electric field was represented by applying a prescribed potential to the upper boundary of the computational domain while the ground was maintained at zero potential. For a uniform background electric field E_{bg} , the applied potential difference was determined by

$$\Delta V_{bg} = E_{bg} \cdot H_d$$

Where H_d is the height of the computational domain. The grounded rocket-triggering wire distorted the background electric field and produced a local field enhancement near the rocket tip.

2.2. Two-Dimensional Axisymmetric Electrostatic Model of the Rocket-Triggering Wire-Ground System

A finite-element solver was adopted to solve the Laplace equation, and the electric field near the rocket tip was acquired. As shown in Fig. 1(a), a two-dimensional axisymmetric computational model, including the rocket and its trailing wire, was constructed. Since the finite-element method requires a bounded solution domain, a $2000 \text{ m} \times 2000 \text{ m}$ closed region was selected as the computational domain with appropriate boundary conditions imposed. After establishing the geometric model, the rocket

body and its trailing wire were subtracted from the computational domain. The remaining region served as the air domain for solving the Laplace equation. The relative permittivity of the air domain was set to 1. Local mesh refinement was implemented around the rocket body and trailing wire to improve the calculation accuracy of the background electric field near the rocket tip.

The potential on the bottom boundary of the model was set to 0 V to represent the ground potential. As the thundercloud-to-ground gap was approximated with a uniform background electric field [19,20], the top boundary was applied with a potential corresponding to the thundercloud potential, so as to establish the target background electric field across the gap. All the remaining outer boundaries were defined as Neumann boundaries $\vec{n} \cdot \vec{D} = 0$.

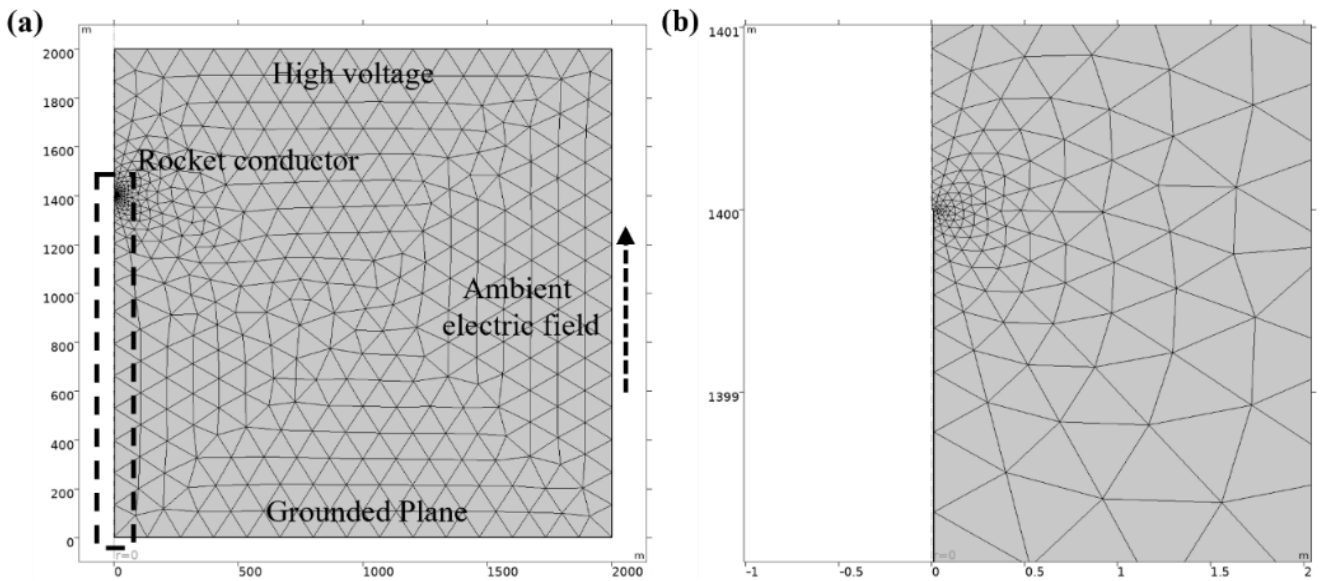


Figure 1. (a) Two-dimensional axisymmetric electrostatic model of the rocket-triggering wire-ground system; (b) Enlarged view of the rocket-tip region in (a)

2.3. Upward Leader Inception Criterion

Several criteria have been proposed for upward positive leader inception from grounded objects, including the critical-radius, generalized leader-inception-voltage, CRFI, and streamer-based physical criteria [21, 22]. According to the CRFI criterion, upward leader inception requires a positive streamer zone of approximately 0.7 m with an electric field of about 500 kV/m [10]. Therefore, a monitoring point was placed 0.7 m above the rocket tip, and upward leader inception is assumed to occur when the electric field at this point exceeds 500 kV/m. For a given cloud-to-ground background electric field, the rocket height at which this condition is first satisfied is defined as the critical rocket height.

3. Results

Figure 2 shows the calculated electric-field distribution for

a rocket height of 1400 m under a cloud-to-ground background electric field of 10 kV/m. As shown in Fig. 2(a), the electric field over most of the computational domain remained relatively low, whereas a pronounced field enhancement was concentrated near the rocket tip. The enlarged view in Fig. 2(b) further showed that the electric field reached the order of $5 \times 10^5 \text{ V/m}$ in the immediate vicinity of the rocket tip and decreased rapidly with increasing distance from the tip. This strong local field enhancement resulted from the distortion of the background electric field by the grounded rocket-triggering-wire system and provided the necessary electrical condition for the inception of upward positive discharges. Therefore, the rocket-tip region was identified as the critical region for evaluating upward-leader inception, and the electric field at 0.7 m above the rocket tip was subsequently used to determine whether the adopted inception threshold of 500 kV/m was satisfied.

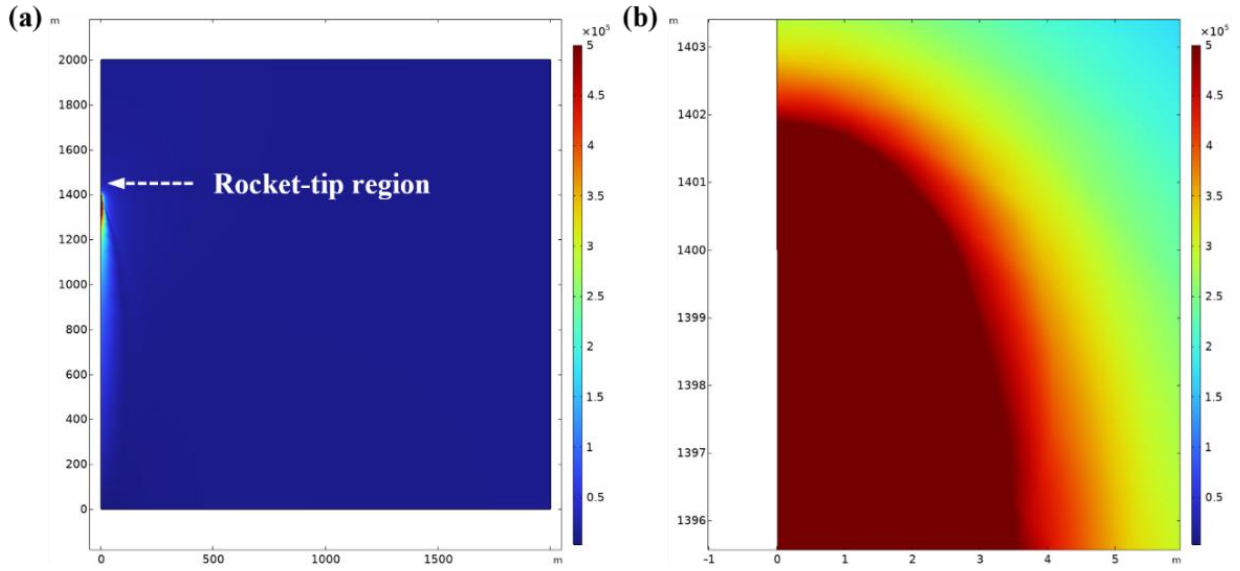


Figure 2. (a) Electric-field distribution at a rocket height of 1400 m under a background electric field of 10 kV/m. (b) Enlarged view of the rocket-tip region in (a)

Field observations have shown that the ambient electric field associated with successful triggering can vary considerably with altitude. Biagi et al. reported ground-level quasi-static electric fields of approximately 3.2–7.6 kV/m before the inception of sustained upward positive leaders, whereas Willett et al. found that lightning could be triggered by grounded wires approximately 400 m long when the ambient electric field aloft was as low as about 13 kV/m [23, 11]. Measurements beneath thunderstorms have also demonstrated that the electric field several hundred meters above ground can be substantially larger than that measured at the surface [24]. Accordingly, 20 kV/m was selected as the upper limit in the present simulations, while the lower limit of 1 kV/m was included to cover a broad range of sub-cloud electrical conditions relevant to rocket-triggered lightning.

As shown in Fig. 3(a), for a given cloud-to-ground background electric field, the electric field at 0.7 m above the rocket tip increased approximately linearly with rocket height, and the rate of increase became larger as the background

electric field increased. Under a relatively weak background electric field of 2 kV/m, the electric field at 0.7 m above the rocket tip remained below the adopted upward-leader inception threshold of 500 kV/m even when the rocket reached 1400 m. In contrast, when the background electric field was 4 kV/m or higher, the 500 kV/m threshold could be reached within the investigated rocket-height range, and the corresponding critical rocket height decreased progressively with increasing background electric field. The same dependence was illustrated in Fig. 3(b): at a fixed rocket height, the electric field at 0.7 m above the rocket tip increased approximately linearly with the background electric field, while a higher rocket position required a lower background electric field to reach the inception threshold. These results demonstrated the combined influence of rocket height and the cloud-to-ground background electric field on the local field enhancement near the rocket tip and indicated that a stronger background electric field substantially reduced the critical rocket height required for upward-leader inception.

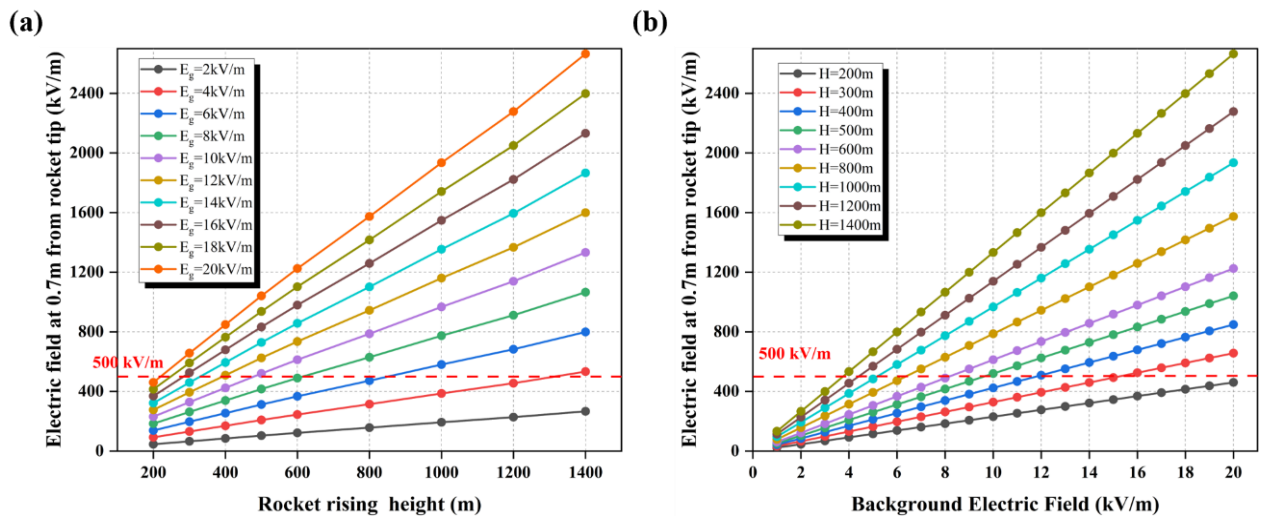


Figure 3. Electric field at 0.7 m above the rocket tip under different rocket heights and background electric fields: (a) variation with rocket rising height under different background electric fields; (b) variation with background electric field at different rocket heights.

4. Conclusion

Determining the critical rocket height is important for evaluating whether the rocket-triggering wire system can satisfy the conditions for upward-leader inception under

different thunderstorm background electric fields. In this study, a two-dimensional axisymmetric electrostatic model was used to investigate the effects of rocket height and cloud-to-ground background electric field on upward-leader inception. The electric field at 0.7 m above the rocket tip

increased approximately linearly with both rocket height and background electric field. Under a background electric field of 2 kV/m, the 500 kV/m inception threshold was not reached even at a rocket height of 1400 m, whereas the threshold could be reached within the investigated height range when the background electric field was 4 kV/m or higher. Overall, increasing the background electric field reduced the critical rocket height required for upward-leader inception.

Acknowledgements

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

References

- [1] Rakov, V. A., Uman, M. A., & Rambo, K. J. (2005). A review of ten years of triggered-lightning experiments at Camp Blanding, Florida. *Atmospheric Research*, 76(1-4), 503–517.
- [2] Qie, X., Jiang, R., & Laroche, P. (2012). Triggering lightning experiments: An effective approach to the research of lightning physics. *AerospaceLab*, 5, 1–12.
- [3] Dwyer, J. R., & Uman, M. A. (2014). The physics of lightning. *Physics Reports*, 534(4), 147–241.
- [4] Gameraota, W. R., Uman, M. A., Hill, J. D., Pilkey, J., Ngin, T., Jordan, D. M., & Mata, C. T. (2013). An “anomalous” triggered lightning flash in Florida. *Journal of Geophysical Research: Atmospheres*, 118, 3402–3414.
- [5] Biagi, C. J., Uman, M. A., Hill, J. D., Rakov, V. A., & Jordan, D. M. (2012). Transient current pulses in rocket-extended wires used to trigger lightning. *Journal of Geophysical Research: Atmospheres*, 117(D7), D07205.
- [6] Biagi, C. J., Uman, M. A., Hill, J. D., & Jordan, D. M. (2011). Observations of the initial, upward-propagating, positive leader steps in a rocket-and-wire triggered lightning discharge. *Geophysical Research Letters*, 38(24), L24809.
- [7] Hill, J. D., Pilkey, J., Uman, M. A., Jordan, D. M., Rison, W., & Krehbiel, P. R. (2012). Geometrical and electrical characteristics of the initial stage in Florida triggered lightning. *Geophysical Research Letters*, 39, L09807.
- [8] Jiang, R., Qie, X., Wang, C., & Yang, J. (2013). Propagating features of upward positive leaders in the initial stage of rocket-triggered lightning. *Atmospheric Research*, 129-130, 90–96.
- [9] Biagi, C. J., Jordan, D. M., Uman, M. A., Hill, J. D., Beasley, W. H., & Howard, J. (2009). High-speed video observations of rocket-and-wire initiated lightning. *Geophysical Research Letters*, 36, L15801.
- [10] Petrov, N. I., & D'Alessandro, F. (2002). Theoretical analysis of the processes involved in lightning attachment to earthed structures. *Journal of Physics D: Applied Physics*, 35(14), 1788–1795.
- [11] Willett, J. C., Davis, D. A., & Laroche, P. (1999). An experimental study of positive leaders initiating rocket-triggered lightning. *Atmospheric Research*, 51(3-4), 189–219.
- [12] Rizk, F. A. M. (1994). Modeling of lightning incidence to tall structures. I. Theory. *IEEE Transactions on Power Delivery*, 9(1), 162–171.
- [13] Petrov, N. I., & Waters, R. T. (1995). Determination of the striking distance of lightning to earthed structures. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 450(1940), 589–601.
- [14] Becerra, M., & Cooray, V. (2006). A simplified physical model to determine the lightning upward connecting leader inception. *IEEE Transactions on Power Delivery*, 21(2), 897–908.
- [15] Becerra, M., & Cooray, V. (2006). Time dependent evaluation of the lightning upward connecting leader inception. *Journal of Physics D: Applied Physics*, 39(21), 4695–4702.
- [16] Bazelyan, E. M., Aleksandrov, N. L., Raizer, Y. P., & Konchakov, A. M. (2007). The effect of air density on atmospheric electric fields required for lightning initiation from a long airborne object. *Atmospheric Research*, 86(2), 126–138.
- [17] Bazelyan, E. M., Raizer, Y. P., & Aleksandrov, N. L. (2007). The effect of reduced air density on streamer-to-leader transition and on properties of long positive leader. *Journal of Physics D: Applied Physics*, 40(14), 4133–4144.
- [18] D'Alessandro, F. (2003). The use of “field intensification factors” in calculations for lightning protection of structures. *Journal of Electrostatics*, 58(1-2), 17–43.
- [19] Mazur, V., & Ruhnke, L. H. (1993). Common physical processes in natural and artificially triggered lightning. *Journal of Geophysical Research: Atmospheres*, 98(D7), 12913–12930.
- [20] Pantuso, J. G., & da Silva, C. L. (2024). Modeling the inception and stepped propagation of upward positive lightning leaders. *Journal of Geophysical Research: Atmospheres*, 129(23), e2024JD041596.
- [21] Becerra, M., & Cooray, V. (2006). A self-consistent upward leader propagation model. *Journal of Physics D: Applied Physics*, 39(16), 3708–3715.
- [22] Cooray, V. (2012). *Lightning electromagnetics*. Institution of Engineering and Technology.
- [23] Biagi, C. J., Uman, M. A., Gopalakrishnan, J., Hill, J. D., Rakov, V. A., Ngin, T., & Jordan, D. M. (2011). Determination of the electric field intensity and space charge density versus height prior to triggered lightning. *Journal of Geophysical Research: Atmospheres*, 116(D15), D15201.
- [24] Soula, S., & Chauzy, S. (1991). Multilevel measurement of the electric field underneath a thundercloud: 2. Dynamical evolution of a ground space charge layer. *Journal of Geophysical Research: Atmospheres*, 96(D12), 22327–22336.