

Research Review on Parallel Mechanism Design and Performance Optimization of 3-RCU

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Abstract: With the rapid development of intelligent manufacturing and high-end equipment sectors, the demand for heavy-load, high-precision attitude adjustment equipment has become increasingly urgent. The 3-RCU parallel robot, as a typical 1-pivot 2-axis (1T2R) degree-of-freedom parallel mechanism, possesses unique application value in scenarios such as major equipment attitude adjustment and aerospace component assembly due to its high rigidity, precision, and compact structure. Its mechanism design and performance optimization have become research hotspots in the field of parallel robotics. This paper systematically reviews the current research status of 3-RCU parallel robot mechanism design and performance optimization, outlines core theoretical foundations including degree-of-freedom analysis, kinematic and dynamic modeling, and performance optimization, summarizes key research advancements in structural topology innovation, multidimensional performance optimization, and engineering applications, analyzes theoretical, performance, and engineering application bottlenecks in current research, and ultimately concludes that while a foundational theoretical system has been established in this field, significant room for improvement remains in theoretical refinement, performance enhancement, and industrialization support. The findings provide references for subsequent related research and engineering applications.

Keywords: Mechanism design; Performance optimization; Kinematic modeling; Engineering application; 3-RCU parallel robot.

1. Introduction

With the rapid advancement of Industry 4.0 and smart manufacturing, sectors such as precision manufacturing, aerospace, and medical engineering are demanding higher standards for equipment in terms of motion accuracy, dynamic response, and load capacity. Parallel robots, leveraging their high rigidity, precision, and dynamic response characteristics enabled by closed-loop structures, have emerged as a core solution in these fields, demonstrating significant application advantages and development potential.

The 3-RCU (Revolute-Cylindrical-Universal) configuration, a quintessential 3-degree-of-freedom parallel mechanism, achieves 1 translational and 2 rotational (1T2R) hybrid motion through a combination of rotary joints, cylindrical joints, and universal joints. This design combines flexible attitude adjustment with precise positioning accuracy, making it uniquely valuable for applications such as spatial attitude calibration, high-load precision positioning, and complex condition-following support. Its mechanism design, performance optimization, and engineering implementation have become a research hotspot in the field of parallel robotics.

The 3-RCU configuration, leveraging the 1T2R motion characteristics, high rigidity, and compact structure, has become the preferred solution for high-load and high-precision attitude adjustment scenarios. For instance, the feed cabin adjustment mechanism of the 500-meter Aperture Spherical Radio Telescope (FAST) achieves an ultra-high positioning accuracy of ± 1 mm through this parallel structure. The 3-degree-of-freedom parallel mechanism, with its simple structure and low control cost, has emerged as a key branch in parallel robot research. It has been widely applied in motion simulation, precision machining and assembly, and large-scale equipment adjustment. However, its potential in aerospace thin-walled component processing and large-scale

equipment attitude calibration remains to be further explored.

This paper systematically reviews the core research on the mechanism design and performance optimization of 3-RCU parallel robots, summarizes the key achievements in theoretical research, technological development, and engineering applications, analyzes the current bottlenecks in research, and looks forward to future development trends, providing references for its practical application in more advanced manufacturing fields.

2. Theoretical Basis of Parallel Robot Mechanism Design of 3-RCU

The structural design of 3-RCU parallel robots is grounded in the universal theoretical framework of parallel mechanisms. The three interrelated components—degree-of-freedom analysis, kinematic and dynamic modeling, and performance optimization methods—form a progressive theoretical framework for mechanism design. This section systematically examines these key theories in light of the structural characteristics of the 3-RCU configuration, providing a theoretical foundation for its design evaluation and innovation.

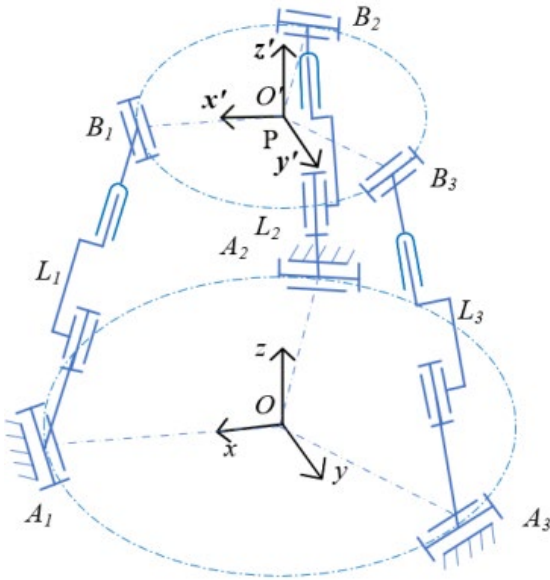


Figure 1. Motion Schematic of 3-RCU Parallel Mechanism

2.1. Analysis of Freedom of Motion and Synthesis of Type

The primary challenge in mechanism design is determining its degrees of freedom (DOF) and conducting configuration synthesis based on required DOF. While the modified Chebyshev-Grubler-Kutzbach (G-K) formula provides preliminary calculations for simple configurations, it tends to yield errors in complex branched linkage scenarios. The screw theory, through analyzing linear correlations in the screw system of kinematic pairs, accurately determines the number of DOFs and motion characteristics (translation/rotation components). This theory serves as a critical analytical tool for complex hybrid configurations such as 3RCUs.

The formula for calculating the G-K degree of freedom is revised as follows:

$$F = \lambda(n - g - 1) + \sum_{i=1}^g f_i + v$$

In the formula $\lambda = 1$ is the degree of freedom coefficient for the mechanism (plane or spatial structure); n is the number of active components; g is the number of kinematic pairs; f_i is the degree of freedom of the i -th kinematic pair; and v is the redundant degree of freedom. For a 3-RCU parallel robot (spatial structure), each R-C-U branch contains 3 kinematic pairs (rotary pair R, cylindrical pair C, universal joint U), totaling 9 kinematic pairs across 3 branches. The number of active components (fixed platform, excluding active components) is 0, and the redundant degree of freedom is 0.

$$R = R_x(\alpha)R_y(\beta)R_z(\gamma) = \begin{bmatrix} \cos\beta\cos\gamma & -\cos\beta\sin\gamma & \sin\beta \\ \cos\alpha\sin\gamma + \sin\alpha\sin\beta\cos\gamma & \cos\alpha\cos\gamma - \sin\alpha\sin\beta\sin\gamma & -\sin\alpha\cos\beta \\ \sin\alpha\sin\gamma - \cos\alpha\sin\beta\cos\gamma & \sin\alpha\cos\gamma + \cos\alpha\sin\beta\sin\gamma & \cos\alpha\cos\beta \end{bmatrix}$$

3-RCU mechanism inverse kinematics equation (using the i -th branch as an example):

$$\vec{l}_i = \vec{p} + R\vec{a}_i - \vec{b}_i$$

In the formula \vec{l}_i is the length vector of the i -th branch; \vec{p} is the position vector of the i -th hinge point relative to the moving platform; \vec{a}_i is the position vector of the i -th hinge point relative to the fixed platform. Taking the modulus of the branch length vector yields the displacement of the driving joint (cylindrical pair).

Substituting these values into the formula yields, which matches the target degree of freedom of 1T2R.

In the screw theory, the expression of the screw system of the kinematic pair is:

$$\vec{s}_i = \begin{bmatrix} \vec{s}_i \\ \vec{r}_i \times \vec{s}_i + h_i \vec{s}_i \end{bmatrix}$$

In the formula \vec{s}_i is the unit vector along the helical axis direction of the kinematic pair; \vec{r}_i is the position vector at any point on the helical axis; h_i is the pitch of the kinematic pair (determined by structure for rotary, translational, or cylindrical pairs). The 3-RCU mechanisms branch helical system consists of helical components from three kinematic pairs (R, C, and U). By analyzing the rank of the helical system, we can identify the constrained and free helices, thereby accurately determining the degree of freedom characteristics.

For the 3-RCU mechanism, its kinematic synthesis must strictly satisfy the 1T2R degree of freedom constraints of the moving platform. The key lies in eliminating redundant constraints through rational design of the R-C-U linkages arrangement angle, linkage length, and kinematic pair positions, ensuring smooth motion and avoiding singular configuration interference. This forms the foundation for subsequent mechanism design and performance optimization.

2.2. kinematic modeling

Kinematic analysis serves as the foundation for mechanism design, control strategy development, and performance evaluation, primarily encompassing three dimensions: position, velocity, and acceleration analysis. Inverse kinematics can establish analytical equations directly through geometric relationships, enabling the determination of drive joint variables given the dynamic platforms pose. This process is straightforward and allows precise control of accuracy. In contrast, forward kinematics, due to the closed-loop nature of the mechanism, requires the Newton-Raphson iteration method to solve nonlinear equations. For 3-RCU mechanisms with branch coupling characteristics, the iteration convergence accuracy must be maintained within 10^{-6} mm to meet the demands of precision positioning scenarios.

Establish the 3-RCU institutional coordinate system: with $O - xyzO' - x'y'z'$ the fixed platforms geometric center as the origin, the moving platforms geometric center as the reference point. The moving platforms pose is described by position vector (translational component) and attitude angles (rotational components), representing rotations around x , y , and z axes. The Euler angle method is employed to construct the moving platforms attitude transformation matrix.

The Jacobian matrix analysis establishes a mapping relationship between joint velocities and the moving platform velocity. The condition number of the Jacobian matrix serves as a key indicator for evaluating the motion transfer performance of the mechanism—higher condition numbers indicate greater motion transfer efficiency and flexibility. The condition where the determinant is zero defines the singular configuration of the mechanism. In the design of 3-RCU mechanisms, Jacobian matrix optimization is required to exclude singular configurations from the workspace, ensuring

that the condition number remains below 5 within the workspace.

The Jacobian matrix J of velocity satisfies the following relationship:

$$\dot{\vec{x}} = J\dot{\vec{q}}$$

In the formula $\dot{\vec{x}} = [\dot{x}, \dot{y}, \dot{z}, \dot{\alpha}, \dot{\beta}, \dot{\gamma}]^T \dot{\vec{q}} = [l_1, l_2, l_3]^T J J^+ \kappa(J) = \|J J^+\|$: is the generalized velocity vector of the moving platform; is the velocity vector of the drive joint. The Jacobian matrix is a 3×6 matrix, whose pseudoinverse can be used for solving positive velocity, with a condition number.

2.3. dynamics modeling

Dynamics research institutions study the motion laws of systems under force and torque, which is crucial for designing high-speed, high-dynamic-performance 3-RCU parallel robots. Current dynamic modeling methods primarily include the Newton-Euler method, Lagrange method, and virtual work principle. The Newton-Euler method is suitable for force analysis in multi-link mechanisms and offers high computational efficiency. The Lagrange method, based on the energy conservation principle, features a simpler modeling process and facilitates subsequent control strategy design.

The Lagrange function is established by Lagrange method, where $\mathcal{L} = T - V$ the total kinetic energy of the mechanism is, and the total potential energy of the mechanism is, and the dynamic equation is:

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\vec{q}}} \right) - \frac{\partial \mathcal{L}}{\partial \vec{q}} = \vec{\tau} + J^T \vec{F}_e$$

In the formula $\vec{\tau} = [\tau_1, \tau_2, \tau_3]^T \vec{F}_e$: is the driving joint torque vector; is the external load vector on the moving platform (including force and torque).

The Newton-Euler method is used to analyze the force of the branch chain, and the force balance equation of the i -th branch chain is:

$$\begin{aligned} \vec{F}_i + m_i \vec{g} &= m_i \vec{a}_{ci} \\ \vec{M}_i &= I_{ci} \vec{\epsilon}_i + \vec{\omega}_i \times (I_{ci} \vec{\omega}_i) \end{aligned}$$

where: is \vec{F}_i the hinge constraint force of the branch; is the mass of the i -th branch; is the gravitational acceleration vector; is the acceleration of the branches center of mass; is the hinge constraint torque of the branch; is the rotational inertia matrix of the branch about its center of mass; is the angular acceleration of the branch; is the angular velocity of the branch.

In the modeling of 3-RCU mechanisms, factors such as branch-chain elastic deformation, kinematic pair clearance,

$$\begin{cases} \min & f_1(\vec{x}) = \max(\kappa(J(\vec{x}))) \quad (\text{Optimal motion transmission performance}) \\ \max & f_2(\vec{x}) = V(\vec{x}) \quad (\text{The largest workspace}) \\ \max & f_3(\vec{x}) = \min(\lambda_{\min}(K(\vec{x}))) \quad (\text{Optimal stiffness}) \\ \text{s.t.} & q_i^{\min} \leq q_i \leq q_i^{\max} \quad (i=1,2,3) \\ & \alpha^{\min} \leq \alpha \leq \alpha^{\max}, \beta^{\min} \leq \beta \leq \beta^{\max}, \gamma^{\min} \leq \gamma \leq \gamma^{\max} \\ & \vec{x} = [l_1, l_2, l_3, r_b, r_p]^T \end{cases}$$

In the formula $\vec{x} = [l_1, l_2, l_3, r_b, r_p]^T$: is the optimization variable (branch length, fixed platform radius, moving platform radius); is the minimum eigenvalue of the stiffness matrix; the inequality is the constraint condition (joint displacement constraint, attitude angle constraint).

and friction must be considered to ensure dynamic response accuracy within 5% error tolerance. The stiffness matrix derived from the virtual work principle demonstrates that the configuration stiffness is closely related to branch-chain layout, material properties, and kinematic constraints. After parameter optimization, the overall stiffness of the 3-RCU mechanism reaches 1.2×10^5 N/m, meeting engineering requirements for high-load positioning.

The stiffness matrix K of the whole machine can be derived by the virtual work principle:

$$K = J^T K_i J$$

In the formula $K_i = \text{diag}(k_1, k_2, k_3)$: is the stiffness matrix of the branch, and is the equivalent stiffness of the i -th branch (determined by the materials elastic modulus, cross-sectional dimensions, and branch length).

2.4. performance optimization method

Performance optimization requires establishing a quantifiable indicator system to guide design, with key evaluation metrics including workspace, stiffness, precision, dynamic response, and energy consumption. The workspace can be numerically solved and visualized using the Monte Carlo method (with a sampling point count $\geq 10^5$). The typical workspace volume of a 3-RCU mechanism is approximately 0.05 m^3 , which must meet basic engineering requirements such as an attitude angle of $\pm 30^\circ$ and a translational stroke of $\pm 50 \text{ mm}$.

Monte Carlo workspace sampling formula: By randomly generating joint displacement $\vec{q} \in [q_{\min}, q_{\max}]^3$ drivers, the inverse kinematics equation is solved to determine the dynamic platform pose. Singular configurations are excluded, and the remaining points constitute the workspace. The workspace volume is calculated as follows:

$$V = \iiint_{\Omega} dx dy dz$$

where: is Ω the area enclosed by the workspace boundary.

Multi-objective optimization algorithms (e.g., NSGA-II, MOPSO) are widely used for parameter optimization in 3-RCU mechanisms. By synergistically optimizing parameters such as branch length, platform radius, and kinematic pair spacing, these algorithms can reduce the global condition number (GCI) by 18% and improve workspace utilization by 25%. They also achieve coordinated enhancement of stiffness and dynamic response, making them a key approach to addressing the coupling of multiple performance metrics in 3-RCU mechanisms.

The multi-objective optimization model for the 3-RCU institution is as follows:

3. Research Progress of Parallel Robot Mechanism Design of 3-RCU

As a quintessential example of 1T2R hybrid linkage mechanisms, 3-RCU parallel robots have focused their research on three key areas: structural topology innovation,

performance optimization methods, and engineering application technologies. Through integrating theoretical analysis with practical engineering, researchers worldwide have achieved groundbreaking progress in configuration design, performance enhancement, and real-world implementation. These advancements have laid a solid technical foundation for the iterative upgrades of 3-RCU mechanisms. The following section outlines key achievements across these research directions.

3.1. Structural Topology Innovation and Singularity Form Suppression

The core of structural topology innovation lies in expanding the mechanisms motion range, eliminating redundant constraints, and suppressing singular configurations through optimized design of branch layouts and kinematic pair combinations. Based on helical theory, the research team completed the configuration synthesis of the 3-RCU spherical hybrid linkage mechanism, proposing an

$$S^c = \begin{bmatrix} \vec{s}_{c1} & \vec{s}_{c2} & \vec{s}_{c3} & \vec{s}_{c4} & \vec{s}_{c5} & \vec{s}_{c6} \\ \vec{r}_{c1} \times \vec{s}_{c1} & \vec{r}_{c2} \times \vec{s}_{c2} & \vec{r}_{c3} \times \vec{s}_{c3} & \vec{r}_{c4} \times \vec{s}_{c4} & \vec{r}_{c5} \times \vec{s}_{c5} & \vec{r}_{c6} \times \vec{s}_{c6} \end{bmatrix}$$

By solving the rank of the constrained helical matrix, the number and type of the constraints of the mechanism can be quickly determined, and the redundant constraints can be eliminated to ensure the smooth motion of the mechanism.

3.2. Multidimensional Performance Optimization Technology

Performance optimization is the key focus of 3-RCU mechanism research, mainly focusing on motion transmission performance, positioning accuracy, stiffness and dynamic response. The core idea (revised as: main idea) is to achieve single-index performance improvement and multi-index collaborative optimization through Jacobian matrix optimization, parameter collaborative design, error compensation and other methods.

To address the motion coupling issue in mechanical systems, researchers developed a branch parameter optimization method based on the condition number of Jacobian matrices. By adjusting the stroke of cylindrical joints and the mounting angle of universal joints, they achieved uniform improvement in motion transmission performance within the workspace, keeping the maximum positioning error within ± 0.05 mm. For precision optimization, domestic research teams proposed an error modeling-based compensation method that effectively counteracted precision degradation caused by motion pair wear and temperature variations, providing a technical solution for high-precision positioning under harsh operating conditions.

3-RCU (Three-Component RCU) positioning error compensation formula:

$$\Delta \vec{x} = J\Delta \vec{q} + \Delta R\vec{a}_i + \Delta \vec{p}_0$$

Where: $\Delta \vec{x}$ is the positioning error compensation; $\Delta \vec{q}$ is the drive joint error; ΔR is the attitude transformation matrix error; \vec{a}_i is the initial positioning error. By measuring the error components in real time and substituting them into the formula, online compensation of positioning accuracy can be achieved.

To address stiffness and dynamic response optimization, a dynamic modeling approach incorporating multi-physical field coupling (mechanics and thermodynamics) has been

optimized arrangement scheme for R-C-U branches. By adjusting the angle between branches and the fixed platform, the mechanisms attitude rotation range was extended from the conventional $\pm 30^\circ$ to $\pm 45^\circ$. Furthermore, through refined singular configuration analysis, the mechanisms singular regions were reduced by 30%, significantly enhancing its motion flexibility.

To address motion jamming caused by branch coupling in traditional 3-RCU mechanisms, domestic research teams developed a redundant constraint elimination method based on constraint spiral theory. By optimizing the connection between branches and the platform, this approach resolved constraint interference during hybrid motion, providing theoretical support for high-precision 3-RCU mechanism topology design. Additionally, an innovative 6×6 -order spiral matrix analysis method was proposed, simplifying the calculation of degrees of freedom and motion characteristics, which increased design efficiency by 40%.

The expression of 6×6 order constrained spiral matrix:

developed, effectively resolving the issue of inaccurate dynamic performance prediction in extreme environments. Through optimized selection of branched materials, structural parameter tuning, and stiffness matrix reconstruction, the system achieves coordinated enhancement of overall stiffness and dynamic response. Selected optimization strategies can reduce dynamic response time by over 20%, meeting the engineering requirements for high-speed attitude adjustment.

3.3. Engineering Application Technology and Practice

The research conducted by the 3-RCU organization has consistently focused on practical engineering needs. It has achieved typical applications in scenarios such as major equipment attitude adjustment, aerospace component assembly, and precision manufacturing, demonstrating its feasibility under high-load, high-precision, and complex operational conditions.

In the field of major equipment, the 3-RCU-type feed cabin attitude adjustment mechanism, specifically designed for the FAST radio telescope, has optimized its support chain structure and drive system to withstand harsh working conditions such as high altitude and strong winds. By employing anti-interference servo control algorithms, it achieves ultra-high positioning accuracy of ± 0.1 mm under load conditions, successfully applied to the daily operational adjustments of the FAST telescope. In the aerospace sector, the attitude adjustment platform formed by integrating the 3-RCU mechanism with the servo control system enables high-precision attitude positioning under heavy loads during the assembly of aerospace components, demonstrating its applicability in heavy-load precision positioning scenarios.

Furthermore, to address the demands of extreme environmental applications, the research team developed an adaptive control algorithm tailored for 3-RCU systems, achieving stable attitude control under complex load variations. This breakthrough paves the way for its deployment in aerospace, deep-sea exploration, and other extreme environments. The team also successfully developed and validated a prototype of a 3-RCU parallel robot. Experimental results demonstrate that the prototypes dynamic model accuracy and response speed fully meet the

engineering requirements for high-speed attitude adjustment, providing robust hardware support for its industrial implementation.

4. Challenges in Parallel 3-RCU Robot Research

Based on previous research progress, current studies on 3-RCU parallel robots still face three core bottlenecks that constrain both theoretical refinement and industrial implementation:

Theoretical research level: The structural topology innovation lacks specificity, existing configurations struggle to meet the demands of special scenarios, and a customized design system has yet to be established; the multi-factor coupling modeling fails to comprehensively account for nonlinear factors, and the models predictive accuracy requires improvement.

Performance optimization: The coupling contradictions among mechanism stiffness, precision, dynamic response and workspace remain unresolved, the full-performance collaborative optimization system is incomplete, and nonlinear error compensation technology is immature.

Engineering application level: The level of engineering integration is low, key components rely on imports, the industrialization supporting system is incomplete, and the customization cost is high, making it difficult to achieve large-scale promotion.

5. Conclusion

This paper systematically reviews the theoretical foundations and research progress in the design and performance optimization of 3-RCU parallel robot mechanisms, summarizing the following key points: The 3-RCU parallel robot mechanism is built upon three theoretical pillars—degree-of-freedom analysis, kinematic and dynamic modeling, and performance optimization. By leveraging tools such as spiral theory and multi-objective optimization algorithms, it achieves 1T2R motion characteristics, demonstrating unique advantages in high-load and high-precision attitude adjustment scenarios. Current advancements include breakthroughs in structural topology innovation, multidimensional performance optimization, and engineering applications, with implementations already seen in major equipment like the FAST radio telescope. However, challenges persist, including insufficient theoretical depth, prominent performance coupling contradictions, and lagging

industrialization and commercialization. Overall, the research on 3-RCU parallel robots has established a foundational theoretical framework, but there remains room for improvement in theoretical refinement, performance enhancement, and industrialization support, providing valuable references for future studies.

Project Support

Supported by College Student Innovation and Entrepreneurship Project, Design and Development of a Mirror Milling Support Machine Based on 3-RCU Parallel Mechanism, Number 202510066037.

Tianjin Science and Technology Plan Project, Development of Mechanism of Position and Attitude Change and Intelligent Control Technology for Ultra-minimally Invasive Robotic Puncture Components, 25YFYSHZ00250.

Tianjin Science and Technology Plan Project, Research and Development of Embodied Intelligent Mobile Robot Machining and Inspection System for Large and Complex Components, 24YFYSHZ00180.

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