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# Unstructured Grid-Based Finite Element Forward Modeling Method for Electromagnetic Fields

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**Abstract:** In complex terrain modern day geophysical electromagnetic work or microwave engineering needs a good model of the electro magnetic field to succeed. The outdated numerical calculations relying equally distributed reguar grid finite diff, and un-even grounds like waves, faults, all sorts of messed up 3D anomalous distortions, it would greatly warp the local EM field response data points in ALL but maybe some last steps before 'inverted', and trying to interpret that result. In order to solve the bottleneck problem, we explore and realize a 3D vector finite element forward modeling method of electromagnetism field with unstructured tetrahedron grids. Utilizes very closely conformed unstructured grids to closely approximate complicated geological model and also has vector basic function based on edge element to remove these kinds of pseudo solution that you have when you do normal node FEM method. This article describes how to obtain the weak form integral equation of Maxwell's equation of the electric field curl and introduces a pre-process Krylov subspace iteration solution strategy for solving big sparse complex linear equations. It is also the proof for truth of both its accuracy and strength in converging, it has experienced a lot of easy numerical check upon simple halfspace, roughest terrain and even most complicated bodies with deeply buried 3D high conductivity targets. The article can be considered true 3dfoward Model Theory Support and algorithm Basis To Get True Data Refinements And Exact Location Of Any Complex Geologic Body Within China while carrying out an Electromagnetic Investigation.

**Keywords:** Unstructured Grid, Electromagnetic Field Forward Modeling, Finite Element Method, Vector Edge Element.

## 1. Introduction

China's energy resource exploration is developing towards deeper and more complex environments, and deep geophysical exploration is an important part of the national major scientific and technological development strategy. Electromagnetic exploration plays an irreplaceable role in the investigation of metallic minerals, deep geothermal resources, oil and gas structures, and hydrogeological engineering because it is very sensitive to differences in underground medium conductivity. Essentially, it is about inferring the electrical properties of underground media by observing the surface or subsurface responses to natural or artificial electromagnetic fields.

## 2. Theoretical Foundation of Electromagnetic Field Finite Element

### 2.1. Maxwell's Equations and Boundary Value Problem

The propagation of electromagnetic fields in underground media is strictly governed by the classical Maxwell equations [1]. Therefore, we can safely ignore the displacement current term in Maxwell's equations and adopt the quasi-static approximation theory. In the frequency domain characterized by angular frequency $\omega$ and time factor $\exp(i\omega t)$ (where $i$ is the imaginary unit), the Maxwell curl equation can be expressed as: $\nabla\times E = -i\omega\mu H$, and $\nabla\times H = \sigma E + J_s$. Here, E represents the electric field intensity vector, H represents the magnetic field intensity vector, $\mu$ is the dielectric constant, $\sigma$ is the dielectric conductivity, and $J_s$ is the external excitation source current density vector. To derive the governing equations required for the finite element method, we take the curl operation on both sides of the first equation and substitute the second equation into it, and after simplification, we can obtain the vector Helmholtz equation satisfied by the electromagnetic field in the frequency domain: $\nabla\times(\nabla\times E) + i\omega\mu\sigma E = -i\omega\mu J_s$. This partial differential equation forms the theoretical starting point for finite element forward modeling on unstructured grids. To ensure the uniqueness of the solution to this partial differential equation, appropriate boundary conditions must be imposed on the boundaries of the truncated computational domain. Typically, on the truncated computational boundaries far from the sources and anomalies, it is assumed that the electromagnetic field has decayed to zero, thereby imposing homogeneous Dirichlet boundary conditions of the first kind, that is, $n\times E = 0$, where n is the unit outward normal vector of the boundary.

### 2.2. Variational Principle and Weak Form Integral Equation

The main idea of the finite element method is to not solve that complex partial differential equation directly but rather change it into its corresponding variational problem or a weighted residual weak form for solution [3]. This paper strictly adopts the weighted residual method (Galerkin method) to derive the weak form integral equation of the electric field double curl equation. We take the inner product of the electric field E to be the arbitrary vector test function V and integrate over the entire computational region $\Omega$: $\int_\Omega [\, V\cdot(\nabla\times(\nabla\times E)) + i\omega\mu\sigma(V\cdot E) \,]\, dV = \int_\Omega V\cdot(-i\omega\mu J_s)\, dV$. Using the first Green identity (divergence theorem) in vector analysis, we can perform partial integration and reduce the second-order derivative term containing the curl operator, converting it into an inner product form of first-order derivatives. Through rigorous mathematical derivation, the above integral equation is rewritten as: $\int_\Omega [\, (\nabla\times V)\cdot(\nabla\times E) + i\omega\mu\sigma(V\cdot E) \,]\, dV - \int_\Gamma V\cdot(n\times(\nabla\times E))\, dS = -i\omega\mu \int_\Omega V\cdot J_s$

dV. Since on the truncation boundary $\Gamma$, the tangential component of the test function V is forced to be zero (satisfying the boundary condition $n \times V = 0$), the area integral term in the above equation automatically disappears. Finally, we obtain the weak form integral equation applicable to the finite element discretization of the electromagnetic field: $\int_\Omega [ (\nabla\times V) \cdot (\nabla\times E) + i\omega\mu\sigma(V \cdot E) ]\, dV = -i\omega\mu \int_\Omega V \cdot J_s\, dV$. This weak form significantly reduces the continuity requirements of the electric field basis functions, laying a solid theoretical foundation for the subsequent introduction of vector edge elements [4].

# 3. Non-structured Grid Generation and Vector Basis Function

## 3.1. Unstructured Tetrahedral Grid Partitioning Technique

To make full use of all the advantages of the finite element method when dealing with complicated geology, good unstructured grid division technology is a must. Compared with structure grids, unstructured tetrahedral grids have very great topological flexibility [5]. Do not need to keep the same node connection pattern all over the computational area: The real forward modeling of electromagnetic fields use an open sourced Grid Generation tool which is either Delaunay Triangulation, AFM to generate the discrete model of geology.

## 3.2. Construction Mechanism of Vector Edge Element Basis Functions

In the traditional scalar finite element method, the unknowns in the degree of freedom are generally defined on each node of the mesh, and then the electric field is interpolated with the nodal basis function [6]. the length of this edge is Lk; Then we can get a vector basis function Nk from it like this: $N_k = L_k (L_i \nabla L_j - L_j \nabla L_i)$, where $L_i$ and $L_j$ are are the dimensionless volume coordinate function (shape function) corresponding to node I and node J. The clever construction of the basis function $N_k$ has two things about mathematics: first is that when we do the tangent projection of the basis function onto itself at edge k, it will be 1, and at another five edges of the tetrahedron, the projection result should all be 0, which gives us a very clear meaning in physics; secondly, its value can also be calculated by the curl property of the basis function, however, the divergence of the basis function is always zero, ($\nabla\cdot N_k = 0$). This “diverging but not diverging” feature can automatically meet the requirement in physical laws that the divergence of electromagnetic waves in the induced area is always 0, thus eliminating the generation of false solutions caused by mathematical reasons. In addition, the edge element basis function just guarantee that the tangential component of electric field have a continuous passing thorough for common surface between adjacent elements, meanwhile allow its normal component can jump freely, which completely satisfy all kinds of complicated requirement of physical boundary condition in electromagnetic medium interface.

# 4. Solving Big Sparse System of Equations Algorithms

## 4.1. Assembly of the Stiffness Matrix and CSR Compressed Storage

After we discretize the integral form weak form with vectors edge elements, the whole continuous PDE problem has been converted to a huge complex linear system which still needs solving and its standard matrix form can be written as $K*x=b$. K is the whole stiffness matrix, including all information about geometries and medium regarding to the EM fields’ propagation and attenuation, x is a complex column vector formed by electric field tangential DOF over all grids edges, b is the load column vector resulting from external excitations(source such as long wire source or magnetic dipole sources) applied on each corresponding edge. If we still use the conventional 2D array to store such matrix, the memory on our computer would be fully occupied, which is simply unacceptable when doing engineering work. So this paper will use the csr sparse row (CSR) format to store all of the stiffness Matrix K [7].

## 4.2. Preprocessed Krylov Subspace Iterative Solving Method

After getting that highly sparse big complicated equation $K*x=b$, how do we solve it quickly but securely turns out to be the final most part after all the preparations for doing 3D FEM forward simulation [8]. For some problems with over ten million degrees of freedom, this method is often unable to handle such problems. In order to break through this bottleneck, we adopt other modern iteration methods based on Krylov Subspace, like Stable (BiCGSTAB), GMRES (Generalized Minimal Residual), etc. The above iteration algorithm is just needed to get the value of large sparse matrix * vector (SpMV) when solving, without changing the structure of sparse matrix at all, so it will use very less memory. In order for the system of equations’ solution to converge faster we need powerful preconditions. In this paper before iterating over the solution we apply left preconditioning on our original equation to get $(M^{-1} * K) * x = M^{-1} * b$. The preconditioner M has been set to be the ILU form of the initial matrix K. The ILU preconditioner creates a nearly inverse matrix for K by omitting the very tiny absolute values from the generated injected element through LU decomposition and greatly improves the spectral distribution characteristics of the initial coefficient matrix so that the eigenvalue distribution characteristic of the processed coefficient matrix will have its eigenvalues much more concentrated near the point (1, 0).

# 5. Modeling Test and Numerical Simulation Analysis

## 5.1. By Applying An Unstructured Grid and Vectors of Edge Elements, It Is Certain That a Three Dimensional FEM Forward Model Algroithm is Right

To verify the accuracy of the three-dimensional finite element forward modeling algorithm based on unstructured grid and vector edge elements proposed in this paper, a classic uniform half-space geoelectric model was designed for comparative analysis. The model sets the electrical conductivity of the air layer to 10^-8 S/m, and the electrical conductivity of the uniform half-space medium underground to 0.01 S/m (i.e., resistivity 100 Ω·m). he specific experimental data is shown in Table 1. This fully proves the rigor of the algorithm theory and the correctness of the programming implementation.

**Table 1.** Comparison of relative error between analytical solution and numerical solution for a uniform half-space model

| Emission Frequency | Number of tetrahedral elements | Degrees of Freedom (DoF) | Analytical solution amplitude (V/m) | Numerical Solution Amplitude (V/m) | Relative Error (%) |
|---|---|---|---|---|---|
| 0.1 | 254,320 | 302,115 | $1.254\times10^{-7}$ | $1.261\times10^{-7}$ | 0.558 |
| 1.0 | 482,150 | 563,890 | $2.845\times10^{-8}$ | $2.852\times10^{-8}$ | 0.246 |
| 10.0 | 954,600 | 1,120,500 | $4.120\times10^{-9}$ | $4.125\times10^{-9}$ | 0.121 |
| 100.0 | 1,845,200 | 2,154,300 | $3.015\times10^{-10}$ | $3.012\times10^{-10}$ | 0.099 |
| 1000.0 | 3,520,100 | 4,120,450 | $1.150\times10^{-11}$ | $1.149\times10^{-11}$ | 0.086 |

In order to further prove the engineering practicability of our solution algorithm, we also compared the performance index on the same large tetrahedron network model with about 2.15M degrees of freedom and other iterative solutions [9].

**Table 2.** Performance comparison of different iterative solvers (DOF: 2,154,300)

| Solver | Preprocessing | Iterations | Time (s) | Memory (GB) | Convergence |
|---|---|---|---|---|---|
| BiCGSTAB | None | >10000 | — | 1.2 | Not converged |
| GMRES (30) | None | 8540 | 3450 | 2.5 | converged |
| BiCGSTAB | ILU (0) | 325 | 185 | 3.8 | converged |
| GMRES (30) | ILU (0) | 210 | 220 | 4.6 | converged |

## 5.2. Electromagnetic Field Response Simulation in Complex Terrains, Undulations Surface Model

In order to illustrate the absolute advantages of unstructured grids on complicated geometric boundaries, a 3D geological model which is very undulated terrain was created. Mountain peak on the terrain model is 300m high, and there is a canyon in the shape of a V, which is 150 meters deep. Take out the amplitude and phase responses of the horizontal component Ex of the electric field and the horizontal component Hy of the magnetic field respectively, and perform finite-element simulation on all the measurement points. The core data are listed in Table 3. Analyzing the data in Table 3 it is easy to see that at the top of the mountain peak (X=500m), because of the terrain contraction aggregation effect, there is an obvious amplification of electric field amplitude and phase delay distortion. And at the bottom of the canyon (X=1500m) the electromagnetic wave spreads out on two sides, so there are some local troughs in the receiving electric and magnetic field amplitudes. And it is this kind of numerical simulation result, to unveil the classic electromagnetic terrain effect Physical law of "mountain peak convergence and canyon dispersion", as well as verify that based on unstructured grids vector finite element algorithm can extremely sensitively and accurately display all kinds of local electromagnetic field anomaly caused by small terrain change, and fully overcome the step-type false anomaly interference in structured grid [10].

**Table 3.** Electric and magnetic field component responses over undulating terrain

| Measurement Point X | Coordinate (m) Terrain | Elevation Z (m) | Geomorphological Features | Ex Amplitude (mV/km) Ex | Phase (°) Hy Amplitude (nT) | Hy Phase (°) |
|---|---|---|---|---|---|---|
| 0 | 0 | distal flat area | 15.24 | –45.2 | 0.420 | –15.1 |
| 250 | 150 | uphill section of a mountain | 18.55 | –48.6 | 0.435 | –16.8 |
| 500 | 300 | The highest point on the mountain peak | 24.12 | –52.4 | 0.452 | –19.5 |
| 1000 | 0 | flat transition zone | 14.98 | –45.5 | 0.418 | –15.3 |
| 1500 | –150 | The lowest point of the canyon | 9.85 | –38.2 | 0.385 | –12.4 |

## 5.3. 3D Anomaly Model's Resitivity Characteristic and the Adaptive Encryption

Real world geophysical mineral exploration practices for deep, hidden high conductivity ore bodies are one of the main tasks in electromagnetic exploration. Thus, for the uniform layer which has a resistance value of 1000 Ω·m, in order to create this kind of threedimensional lowresistance cube abnormal model, it is 400m × 400m × 200m, and the center buried depth is 500m. To learn more about what difference in anomaly body conductivity contrast affects on observing data from the surface electromagnetics, we have made 3 operating conditions according to multi-frequency EMF FEM forward model by setting anomaly body resistivity value as 10 Ω·m, 1 Ω·m, and 0.1 Ω·m respectively. We calculate and extract the apparent resistivity data from the surface observation points right above the anomaly body which is converted according to the Cagniard formula of the electromagnetic field at different frequency. Experiement results can be seen from table 4.

**Table 4.** Apparent resistivity at the surface observation point above the anomaly body for different conductivity contrasts

| background resistivity(Ω·m) | Abnormal body resistivity (Ω·m) | 1000 Hz visual resistivity (Ω·m) | 100 Hz apparent resistivity (Ω·m) | 10 Hz apparent resistivity (Ω·m) | 1 Hz apparent resistivity (Ω·m) |
|---|---|---|---|---|---|
| 1000 | 1000(No abnormalities) | 1000.0 | 1000.0 | 1000.0 | 1000.0 |
| 1000 | 10 | 998.5 | 845.2 | 210.4 | 654.8 |
| 1000 | 1 | 998.1 | 780.6 | 85.2 | 412.3 |

According to the data pattern in Table 4, we can deeply understand the characteristic of skin effect of electromagnetic field in three dimensional conductive medium. This entire evolution pattern of the wideband apparent resistivity depth profile is very consistent with the classic theory of 3D electromagnetics, it can provide good forward model samples to support the accurate inversion and location of 3D hidden ore bodies in the frequency range.

**Table 5.** Comparison of calculation accuracy and computational cost before and after local grid adaptive refinement

| Grid Strategy | Maximum Unit Size of Anomalous Body (m) | Total degrees of freedom | E Field Residual (%) | H Field Residual (%) | Time Consuming (s) |
|---|---|---|---|---|---|
| Global fine grid uniform coarse grid of boundary adaptive encryption grid | 150 | 35.0ten thousand | 8.54 | 9.12 | 45 |
| Global fine grid uniform coarse grid of boundary adaptive encryption grid | 20 | 845.0ten thousand | 0.85 | 0.92 | 14500 |
| Global fine grid uniform coarse grid of boundary adaptive encryption grid | interior, inside, inward, indoor, bosom 20, External Gradient Stretch | 125.1ten thousand | 1.05 | 1.14 | 125 |

# 6. Conclusion

In order to meet the requirements of complex deep geophysical exploration and engineering electromagnetics applications, this paper designs a three-dimensional electromagnetic field vector finite element forward modeling system based on unstructured tetrahedral meshes. This system replaces the traditional scalar nodal finite element method with curl-free and active vector edge element basis functions, which can effectively suppress spurious solutions arising from physical laws during electromagnetic numerical calculations, and can also alleviate the discontinuity of the normal components of the electromagnetic field caused by abrupt changes in the properties of geological interfaces. Unstructured tetrahedral fine meshes combined with targeted local adaptive refinement strategies have good adaptability, allowing them to accommodate complex mountainous terrain, irregular boundaries, and the morphological characteristics of complex three-dimensional concealed ore bodies, thereby meeting the requirements of simulating various complex geological models.

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