International Journal of Advanced Engineering and Technology Research
ISSN: 3106-8677 (Print), ISSN: 3106-8685 (Online) | Vol. 1, No. 2, 2026

An Electro-Thermal Battery Runtime Prediction Framework Based on Markov Chain and Monte Carlo Simulation

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Abstract: This paper proposes an electrothermal coupling modeling and stochastic simulation method for predicting battery runtime in mobile devices. An electrical model of lithium-ion batteries is constructed based on Thevenin equivalent circuits, combined with SOC dynamics and polarization voltage evolution to establish a continuous-time state-space model. Concurrently, thermal equilibrium equations describe Joule heating generation and convective heat dissipation, forming an electro-thermal coupled kinetic model to characterize the coupling relationships among temperature, current, and internal resistance. For load modeling, a Markov chain describes the random switching of device operating states, overlaid with Gaussian perturbations to construct a stochastic power model. Based on this, Monte Carlo simulations generate multiple load sequences, iteratively updating SOC, voltage, and temperature states to derive the statistical distribution of the device's remaining operational time. Results demonstrate that this method effectively reflects the impact of multiple factors on battery endurance performance, exhibiting good applicability and potential for broader implementation.

Keywords: Markov Chain, Electro-Thermal Coupled Model, Battery Runtime Prediction.

1. Introduction

As the performance of smart mobile devices continues to improve, their computational capabilities and application scenarios are constantly expanding. Battery endurance has gradually become a critical factor affecting the user experience. Accurately predicting remaining battery runtime not only helps optimize device energy consumption management but also holds significant importance for designing power management strategies and system scheduling. However, in real-world usage scenarios, device load typically exhibits pronounced random fluctuations. Simultaneously, the battery's internal state is influenced by multiple factors including temperature variations, polarization effects, and aging. This convergence of factors transforms runtime prediction into a complex dynamic modeling challenge.

Existing research predominantly relies on simplified electrochemical models or empirical statistical methods for estimation. One approach focuses on establishing empirical relationships based on experimental data collected under fixed load conditions, yet struggles to capture dynamic variations in complex scenarios. Another approach employs data-driven models for prediction, yet often neglects the coupling mechanism between internal electrical and thermal processes, limiting the model's generalization capability across different operating environments. Therefore, constructing a unified modeling framework that simultaneously captures both the physical mechanisms of batteries and load randomness has become a crucial direction for enhancing prediction reliability [1].

To address these challenges, this paper proposes a battery runtime prediction method integrating electrothermal coupling modeling with random load description. First, an electrical model based on equivalent circuits is established, incorporating thermodynamic equations to construct an electrothermal-coupled state-space framework [2]. Subsequently, Markov chains describe the random switching of device usage scenarios, while random perturbations characterize power fluctuation patterns. Building upon this foundation, multiple load sequences are generated through Monte Carlo simulations [3]. The evolution of the battery state is then numerically solved to obtain statistical estimates of the device's remaining operational time. Using mobile terminal devices as a case study, the proposed model demonstrates its applicability under diverse operating conditions, providing a versatile algorithmic framework for predicting battery runtime in complex load environments.

2. Electro-Thermal Battery Modeling Method

2.1. Electrical Equivalent Model

A dynamic lithium-ion battery model is developed based on an equivalent circuit representation. The internal electrochemical behavior of the battery is approximated using a Thevenin-type circuit composed of an ideal voltage source V_{OCV}, a series ohmic resistance R_0, and a parallel RC network consisting of a polarization resistance R_p and a polarization capacitance C_p. This configuration provides an effective approximation of the external voltage characteristics while maintaining computational tractability [4].

2.2. State of Charge Dynamics

The State of Charge (SOC) represents the ratio between the remaining available capacity and the nominal capacity of the battery. Its temporal variation is determined by the discharge current. Let Q_{rated} denote the rated battery capacity and $I(t)$ the load current. The SOC dynamics are described by

$$\frac{dSOC(t)}{dt} = -\frac{I(t)}{3600Q_{rated}} \quad (1)$$

Where the factor 3600 converts ampere-hours to seconds.

2.3. Polarization Dynamics and Terminal Voltage

During battery operation, electrochemical polarization introduces a delayed voltage response. The polarization voltage $V_p(t)$ follows a first-order dynamic process

$$\frac{dV_p(t)}{dt} = \frac{I(t)}{C_p} - \frac{V_p(t)}{\tau_p} \quad (2)$$

Where $\tau_p = R_p C_p$ denotes the polarization time constant, characterizing the transient response speed.

The terminal voltage of the battery is obtained by subtracting internal voltage losses from the open-circuit voltage

$$V_{term}(t) = V_{OCV}(SOC) - I(t)R_0(T, SOC) - V_p(t) \quad (3)$$

Where $V_{OCV}(SOC)$ represents the open-circuit voltage as a function of SOC, typically obtained through experimental calibration. The ohmic resistance R_0 depends on both temperature and SOC.

In modern smartphones, the internal power management circuit regulates the device operating conditions. Consequently, the device load can be approximated as a constant-power demand $P_{load}(t)$. Under this assumption, the relationship between current and terminal voltage becomes

$$I(t) = \frac{P_{load}(t)}{V_{term}(t)} \quad (4)$$

Substituting the terminal voltage equation yields the implicit power balance relation:

$$P_{load}(t) = I(t)\left[V_{OCV}(SOC) - V_p(t) - I(t)R_0(T)\right] \quad (5)$$

2.4. Thermal Dynamics Model

Battery temperature evolution results from the balance between internally generated heat and heat dissipation to the surrounding environment. The thermal behavior can be described as

$$mC_h \frac{dT(t)}{dt} = Q_{gen}(t) - Q_{diss}(t) \quad (6)$$

Where m is the battery mass and C_h is the specific heat capacity.

The heat generated by Joule heating is

$$Q_{gen}(t) = I(t)^2 R_{total}(T) \quad (7)$$

Heat dissipation occurs through convective heat transfer at the battery surface

$$Q_{diss}(t) = hA\left(T(t) - T_{amb}\right) \quad (8)$$

Where h denotes the convective heat transfer coefficient, A is the effective heat dissipation area, and T_{amb} represents the ambient temperature.

The temperature dependence of internal resistance follows an Arrhenius-type relationship

$$R_{total}(T) = R_{ref} \exp\left[\beta\left(\frac{1}{T(t)} - \frac{1}{T_{ref}}\right)\right] \quad (9)$$

Where R_{ref} is the reference resistance at temperature T_{ref} and β is the thermal activation coefficient determined by battery material properties.

The electrical and thermal models together form a coupled electrothermal system describing the battery discharge dynamics.

2.5. Continuous-Time State-Space Representation

To provide a compact mathematical description of the system dynamics, the battery model is formulated in continuous-time state-space form. The model contains three state variables representing charge level, polarization voltage, and temperature.

The state vector is defined as $\mathbf{x} = [SOC(t), V_p(t), T(t)]^T$.

The SOC dynamics follow

$$\frac{dSOC(t)}{dt} = -\frac{I(t)}{3600Q_{rated}} \quad (10)$$

The polarization voltage evolves according to

$$\frac{dV_p(t)}{dt} = \frac{I(t)}{C_p} - \frac{V_p(t)}{R_p C_p} \quad (11)$$

The temperature dynamics are governed by

$$\frac{dT(t)}{dt} = \frac{1}{mC_h}\left[I(t)^2(R_0(T) + R_p) - hA(T(t) - T_{amb})\right] \quad (12)$$

Under a constant power demand P_{load}, the operating current is obtained from the positive real solution of the quadratic equation

$$I(t) = \frac{(V_{OCV} - V_p) - \sqrt{(V_{OCV} - V_p)^2 - 4R_0 P_{load}}}{2R_0} \quad (13)$$

2.6. Runtime Definition

The device runtime, referred to as Time-to-Empty (TTE), is defined as the earliest time when either the charge is depleted or the terminal voltage reaches the cutoff threshold V_{cutoff} (typically 2.8 V):

$$TTE = \min t \mid SOC(t) \le 0 \vee V_{term}(t) \le V_{cutoff} \quad (14)$$

3. Runtime Prediction Method Based on Stochastic Load Modeling

3.1. Stochastic Load Modeling

(1) Markov Chain Representation of Usage Scenarios

The switching behavior of a smartphone among different operating scenarios can be approximated by a discrete-time Markov process. The transition dynamics between usage states are represented by a transition probability matrix [5]:

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1k} \\ p_{21} & p_{22} & \cdots & p_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ p_{k1} & p_{k2} & \cdots & p_{kk} \end{bmatrix} \tag{15}$$

Where $p_{ij} = P(X_{t+1} = s_j \mid X_t = s_i)$ denotes the probability that the device transitions from scenario s_i to scenario s_j at the next time step.

This stochastic representation captures the switching behavior of common user activities such as standby, browsing, multimedia playback, and gaming.

(2) Instantaneous Power Fluctuations

In real usage conditions, device power consumption fluctuates due to background processes, network variations, and user interactions. To represent these fluctuations, Gaussian noise is superimposed on the baseline power consumption of each scenario.

The instantaneous load power is therefore modeled as

$$P_{load}(t) = \mu_{X(t)} + \sigma_{X(t)}\xi(t), \quad \xi(t) \sim N(0,1) \tag{16}$$

Where $\mu_{X(t)}$ and $\sigma_{X(t)}$ denote the mean and standard deviation of power consumption under the current operating state.

(3) Battery Health Degradation

Battery aging influences both the available capacity and the internal impedance.

The maximum releasable charge decreases approximately linearly with the State of Health (SOH):

$$Q_{max}(SOH) = Q_{rated} \cdot SOH \tag{17}$$

Meanwhile, battery aging increases internal resistance. According to empirical degradation models, the total internal resistance can be expressed as

$$R_{total}(SOH, T, SOC) = R_{new}(T, SOC)\left[1 + \gamma\left(\frac{1}{SOH} - 1\right)\right] \tag{18}$$

Where γ is the aging sensitivity coefficient.

The increased resistance amplifies the ohmic voltage drop $V_{drop} = I(t) \cdot R_{total}$, which accelerates the decline of terminal voltage during discharge.

3.2. Runtime Estimation via Stochastic Simulation

Battery runtime is estimated using a Monte Carlo simulation framework [6].

Simulation parameters such as ambient temperature T_{amb}, initial SOC, and SOH are fixed. A total of N stochastic simulations are conducted, typically with $N = 1000$.

For the n-th simulation, a time-varying power sequence $P_{load}^{(n)}(t)$ is generated using the Markov-based load model.

At each simulation step δt, the electrothermal coupled model is updated as follows:

(1) The internal resistance is updated according to the current temperature and SOC.

(2) The instantaneous operating current is obtained from the power balance equation.

(3) SOC, polarization voltage, and temperature are updated using numerical integration methods such as Euler or Runge–Kutta schemes.

(4) The runtime termination condition is evaluated. If satisfied, the runtime TTE_n is recorded.

After all simulations are completed, the runtime samples $TTE_1, TTE_2, \ldots, TTE_N$ are used to calculate the statistical metrics μ_{TTE} and σ_{TTE}, which represent the mean and standard deviation of the predicted runtime.

3.3. Impact of System Parameters on Runtime

(1) Influence of Initial State of Charge

Figure 1 illustrates the relationship between the initial SOC and device runtime.

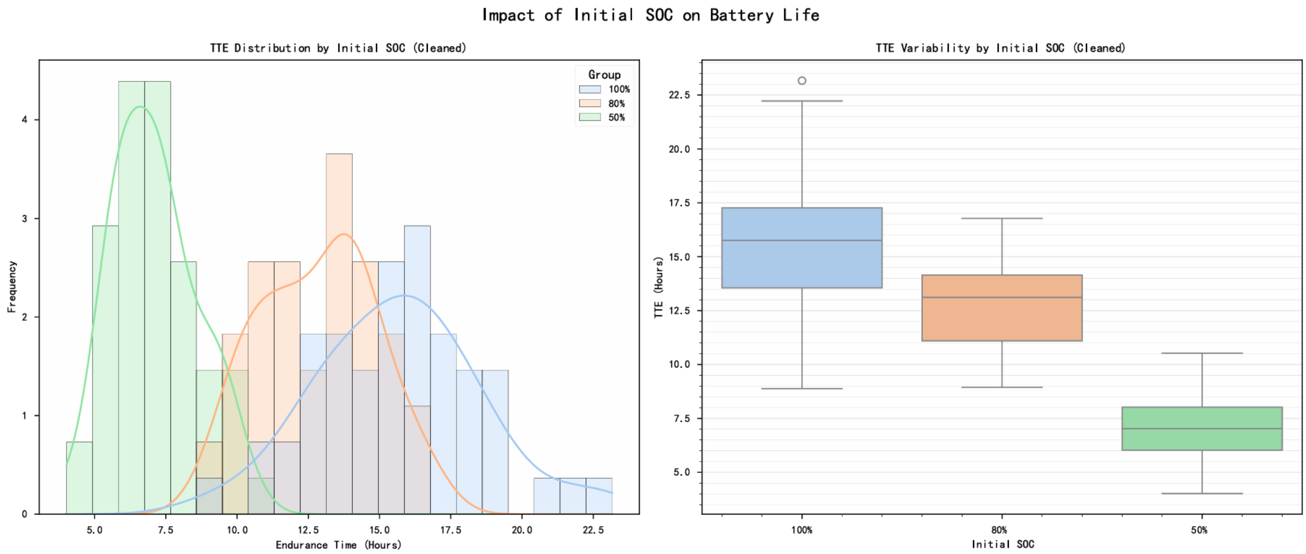

Figure 1. Runtime vs initial SOC

Under fixed conditions ($SOH = 1.0$, room temperature, and standard load), stochastic simulations are performed for three initial SOC levels: 100%, 80%, and 50%.

The results show that runtime is approximately proportional to the initial SOC. Devices starting with full charge achieve the longest runtime, whereas the runtime at 50% SOC is roughly half of the fully charged condition.

The distribution of TTE within each SOC group reflects stochastic load variability. However, the overlap between groups remains limited, indicating that the initial SOC mainly determines the baseline runtime.

From the SOC evolution equation $\frac{dSOC}{dt} = -\frac{I}{3600Q_{rated}}$ when the average current remains stable, the runtime satisfies $T \propto \frac{SOC_0 Q_{rated}}{I}$, which explains the observed proportional relationship.

(2) Influence of Ambient Temperature

The influence of ambient temperature is analyzed under fixed conditions ($SOC = 100\%$, $SOH = 1.0$, standard load).

Three temperature environments are considered: 0°C, 25°C, and 45°C, as shown in the Figure 2.

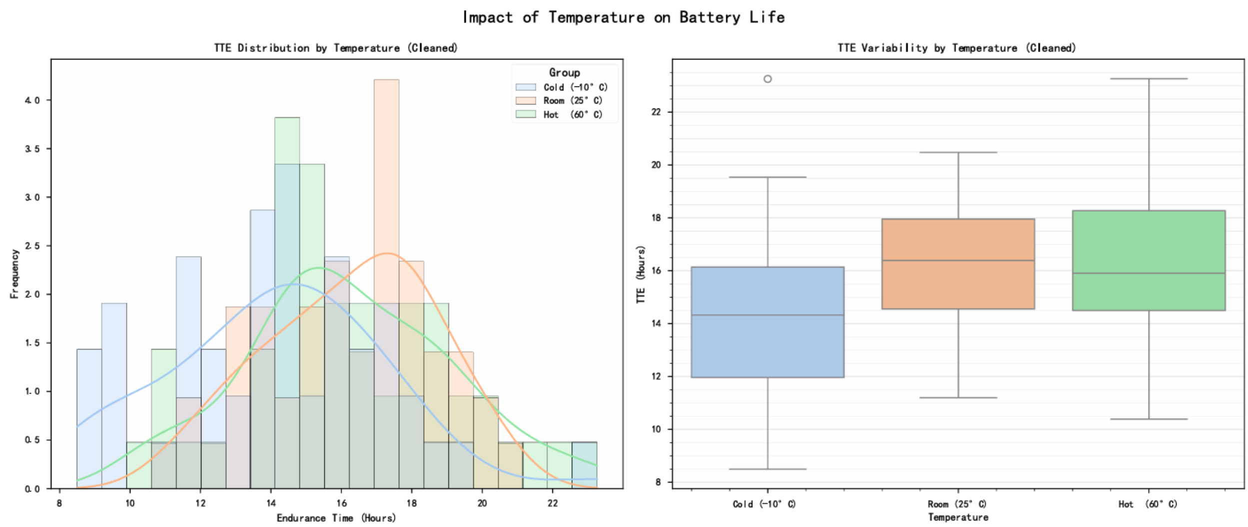

Figure 2. Runtime under different temperatures

The simulation results indicate that low temperature significantly reduces runtime. This effect originates from the Arrhenius dependence of internal resistance.

At low temperatures, the exponential increase in internal resistance leads to increased Joule heat loss I^2R and larger ohmic voltage drop $I \cdot R$, which causes the terminal voltage to reach the cutoff threshold earlier.

High temperatures slightly reduce runtime due to accelerated side reactions and reduced effective capacity.

(3) Influence of Load Intensity

Three workload conditions are considered: power-saving mode (0.7× baseline load), normal mode (1.0×), and high-performance mode (1.5×).

From the Figure 3, the results indicate a strong negative correlation between workload intensity and runtime.

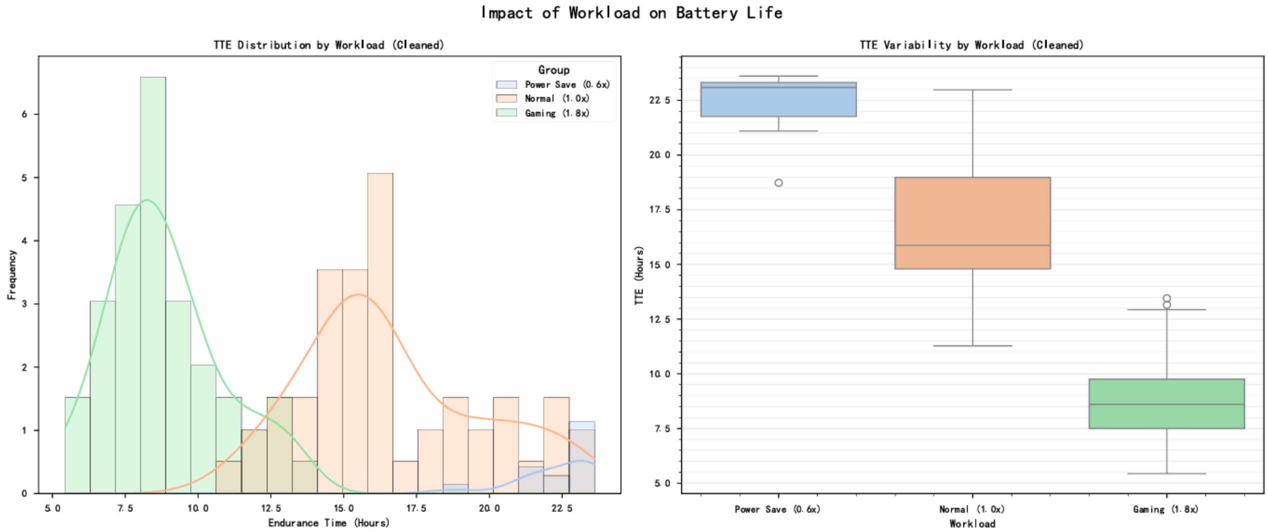

Figure 3. Runtime under different load levels

As the power demand increases, the discharge current increases according to $I = \frac{P}{V_{term}}$, which accelerates energy depletion and shortens the operating time.

3.4. Model Validation

In the Figure 4, the predictive performance of the electrothermal coupled model is evaluated by comparing the simulated SOC evolution under different temperature conditions.

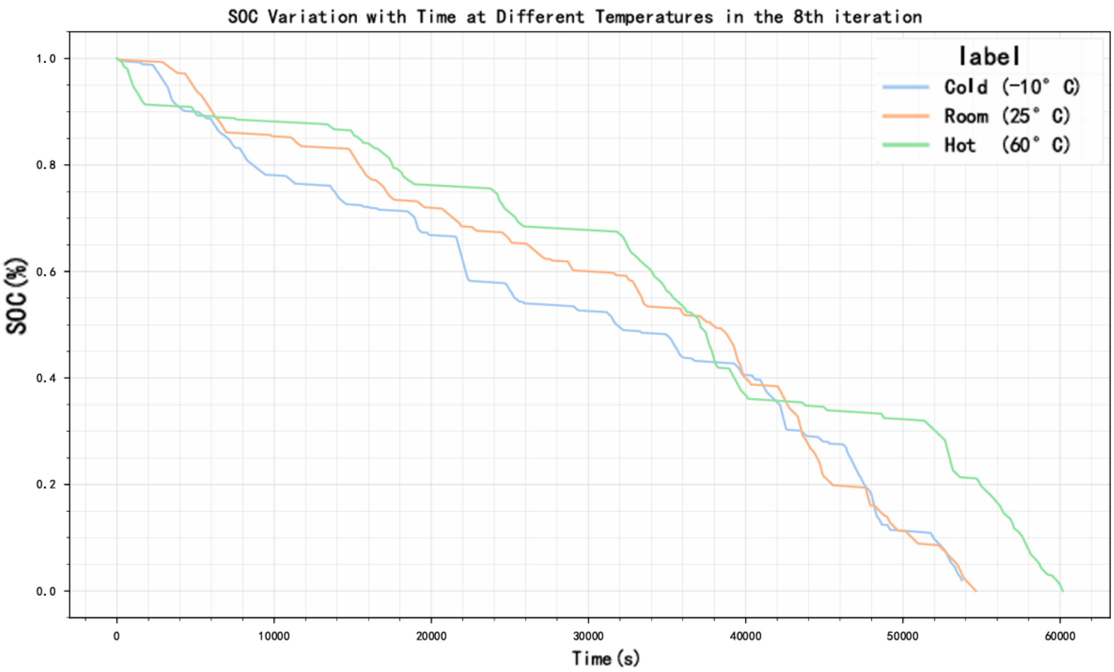

Figure 4. SOC evolution over time

The results demonstrate that low temperature leads to faster voltage decline and earlier runtime termination, whereas room temperature conditions provide longer device operation.

Additionally, nonlinear SOC depletion behavior is observed. When SOC decreases below approximately 20%, the discharge rate increases significantly.

This behavior is explained by the reduction of open-circuit voltage $V_{OCV}(SOC)$ during discharge.

Under constant power demand $I = \frac{P}{V_{term}}$, a decrease in terminal voltage requires higher current, which accelerates charge depletion.

3.5. Model Limitations

Although the proposed model captures the dominant electrothermal mechanisms governing battery discharge, several limitations remain.

First, transient voltage fluctuations under extreme low temperatures and sudden high loads are not explicitly modeled.

Second, battery aging is simplified through the SOH parameter, without distinguishing between calendar agingand cycle aging mechanisms.

3.6. Summary of Influencing Factors

A comparative analysis indicates that load intensity exerts the strongest influence on runtime, followed by battery health (SOH).

Initial SOC plays an important role when the charge level becomes low, while ambient temperature mainly affects performance under cold conditions.

High load, low SOC, degraded battery health, and low temperature together represent the primary conditions that lead to abrupt runtime loss.

4. Conclusion

This paper addresses the challenge of predicting battery runtime for mobile devices by establishing a unified algorithmic framework that integrates electrical models, thermal models, and random load descriptions. A battery electrical model is constructed using a Thevenin equivalent circuit, combined with SOC dynamics and the polarization voltage equation to form a continuous-time state-space representation. Concurrently, a thermal equilibrium mechanism is introduced to characterize the impact of temperature variations on internal resistance and discharge behavior. For load modeling, a Markov chain describes transitions between device operating scenarios, while Gaussian perturbations characterize power fluctuations. Monte Carlo simulations are then employed to statistically estimate runtime. Simulation results demonstrate that the model accurately reflects runtime trends under varying initial SOC, ambient temperatures, and load intensities. These findings provide valuable insights for mobile device endurance assessment and energy management. Future research may further enhance model accuracy by integrating more refined aging mechanisms with real-time data-driven approaches.

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