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Evolution Mechanism of Fluid Flow Regimes under the Dual Constraints of Fracture Geometric Roughness and Aperture Heterogeneity

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Abstract: This study investigates the nonlinear flow behavior in rough fractures under the combined influence of surface roughness and aperture heterogeneity using the lattice Boltzmann numerical simulation method. Surface roughness is decomposed into the local roughness effect, hydraulic tortuosity effect, and surface tortuosity effect. In addition, an aperture roughness effect induced by aperture heterogeneity is introduced. From a mechanistic perspective, the influences of these four effects on fluid flow characteristics within fractures are systematically analyzed. The results show that the Forchheimer equation can adequately describe nonlinear fluid flow in rough fractures, and its nonlinear coefficient can be determined by quantitatively characterizing fracture geometry. Based on the fourfold effects of fracture seepage, the fracture geometric structure is quantitatively described, and the transition mechanism between linear and nonlinear flow under the influence of rough fracture geometry is clarified.

Keywords: Nonlinear flow, surface roughness, lattice Boltzmann numerical simulation method, Forchheimer equation.

1. Introduction

With the rapid development of the economy, the demand for energy has increased significantly. To effectively meet the needs of production and daily life, it is urgent to achieve efficient development and rational utilization of energy resources. Coalbed methane is a clean and renewable energy source. Its efficient development and utilization can not only compensate for the shortage of conventional oil and gas resources, but also reduce production costs and greenhouse gas emissions.

Fractures and their network structures serve as the primary migration pathways for coalbed methane and are regarded as the fundamental research objects for understanding seepage behavior in coal reservoirs. Studies on fracture networks are generally based on single-fracture models. In recent decades, extensive research has been conducted on fluid flow in fractures. However, due to the complex geometric morphology of fractures, it remains difficult to accurately describe the flow process within a single fracture [1, 2]. In early studies, rough fractures with variable apertures were simplified as smooth parallel plates, leading to the well-known cubic law [3]. However, numerous experiments have demonstrated that the cubic law is only applicable to low-velocity laminar flow. Under high hydraulic gradients, a nonlinear relationship between pressure gradient and flow rate emerges, resulting in an overestimation of fracture permeability by the cubic law [4]. This discrepancy is known as the non-Darcy effect, which is attributed to changes in flow direction and velocity induced by fracture surface roughness, contact, and curvature, as well as local eddies and even turbulence.

In the past decades, numerous studies on fluid transport in rough fractures have been reported. The Forchheimer equation provides a fundamental theoretical framework for describing nonlinear flow. However, most studies characterize the influence of fracture surface contact distribution on nonlinear flow using single parameters [5, 6] (e.g., peak roughness or contact area ratio), which is insufficient to reveal the underlying mechanism of nonlinear flow. Meanwhile, the use of a critical Reynolds number to determine the transition from linear to nonlinear flow does not effectively correlate with fracture surface roughness and contact distribution characteristics that induce nonlinear behavior [7-10]. As a result, it is difficult to accurately represent the intrinsic control mechanisms of coalbed methane flow regimes (i.e., linear and nonlinear flow), and this also limits proper calibration, preventing the establishment of reliable theoretical criteria for evaluating coalbed methane production efficiency [11-13].

To address these issues, and for simplification without loss of generality, a two-dimensional rough fracture model is adopted in this study. The lattice Boltzmann method is employed to simulate fluid flow processes. Taking fracture surface geometry as the key controlling factor of fracture flow, this study focuses on the influence mechanisms of surface roughness and contact distribution on fracture flow, including both linear and nonlinear regimes. The constraints and transition mechanisms between linear and nonlinear flow under conditions of fracture roughness and mismatch are investigated, providing further mechanistic insight into complex coalbed methane transport behavior.

2. Theory and Methods

2.1. Nonlinear Seepage Theory

Previous studies have shown that, under isothermal and laminar flow conditions, and based on the smooth parallel-plate model, fluid flow behavior follows the cubic law, i.e., the volumetric flow rate per unit time is proportional to the cube of the fracture aperture [14, 15]:

$$Q = \frac{W\delta^3 \nabla p}{12\mu} \tag{1}$$

In the above expression, Q denotes the volumetric flow rate

per unit time, δ represents the aperture of an ideal smooth fracture, W is the fracture width perpendicular to the flow direction, μ is the dynamic viscosity of the fluid, and ∇p is the pressure gradient.

However, due to the complex geometry of natural fractures, predictions based on the cubic law are often overestimated. As the flow velocity increases, inertial losses may trigger nonlinear flow behavior within fractures. The mechanisms responsible for inertial losses can be attributed to two aspects: (1) changes in flow pathways caused by aperture variations or obstacles, leading to additional inertial resistance; and (2) the development of local vortices and even turbulence induced by continuously increasing flow velocity. The nonlinear flow behavior is commonly described by the Forchheimer equation:

$$-\nabla p = AQ + BQ^2 \tag{2}$$

$$A = \frac{12\mu}{we_h^{\ 3}}, B = \frac{\beta\rho}{w^2 e_h^{\ 2}} \tag{3}$$

In the above equation, A is the linear term coefficient, B is the nonlinear term coefficient, β is the nonlinear resistance coefficient, and ρ is the fluid density. The key aspect of the Forchheimer equation lies in the determination of coefficients A and B. Coefficient A depends on the fracture aperture and reflects the intrinsic permeability of the fracture, whereas coefficient B is related to β.

The Reynolds number, which characterizes the flow regime, is defined as the ratio of inertial forces to viscous forces[16]:

$$\mathrm{Re} = \frac{\rho v e_h}{\mu} = \frac{\rho Q}{\mu w} \tag{4}$$

where v denotes the fluid velocity within the fracture. The critical Reynolds number Rec is commonly used to represent the threshold for the transition from linear to nonlinear flow, and its value may range from 0.001 to 65 or even higher. In addition, the non-Darcy effect factor E is defined as the ratio of the pressure gradient induced by nonlinear flow to the total pressure gradient:

$$E = \frac{BQ^2}{AQ + BQ^2} \tag{5}$$

E is a dimensionless parameter ranging from 0 to 1, representing the intensity of nonlinear flow. A larger value of E indicates a stronger nonlinear flow effect. At present, E = 0.1 is commonly regarded as the threshold between linear and nonlinear flow regimes [17-20]. By combining Eqs. (4) and (5), the critical Reynolds number can be expressed as:

$$\mathrm{Re}_c = \frac{A\rho}{9B\mu w} \tag{6}$$

Equation (6) indicates that a smaller Rec corresponds to a more pronounced inertial effect, and the fluid flow is more likely to transition into a nonlinear flow regime.

2.2. Construction of Two-Dimensional Rough Fracture Composite Topography

In this study, the two-dimensional Weierstrass–Mandelbrot (W-M) function is adopted to characterize the composite topography of rough fractures, ensuring that the generated fracture surfaces exhibit randomness, continuity, and self-affine properties. The W–M function is expressed as follows:

$$Z(x) = \sum_{n=n_l}^{n_h} \gamma^{n(D-2)} \left[\cos\phi - \cos\left(\frac{2\pi\gamma^n x}{L} + \phi\right) \right] \tag{7}$$

In the above expression, Z(x) denotes the fracture surface height; n_l and n_h represent the low-frequency and high-frequency cutoff values, respectively; D is the fractal dimension, where a larger D indicates more pronounced surface roughness, whereas a smaller D corresponds to a relatively smoother fracture surface; L is the overall sample length of the fracture; and ϕ is a random number ranging within $0 \sim 2\pi$.

Brown et al. experimentally observed the mismatch characteristics between the upper and lower fracture surfaces, and further confirmed that the two surfaces are independent at short-wavelength scales while remaining correlated at long-wavelength scales. Based on this observation, our previous work proposed a weighting-based approach to characterize the self-affine mismatch behavior of rough fractures.

The procedure for constructing a two-dimensional rough fracture is as follows. First, two independent random phase sets, $\{\phi_{1,n}\}$ and $\{\phi_{2,n}\}$, are generated, and the upper fracture surface is constructed based on these phases and Eq. (7). Second, according to the mismatch characteristics of natural fracture surfaces, a new random phase set for the lower surface is determined within different wavenumber ranges. Finally, the lower fracture surface is constructed based on the corresponding phase set and Eq. (7). The calculation of this phase set is given by Eq. (8):

$$\phi_m = \begin{cases} \phi_{1,n}, & n_l \le n \le n_1 \\ \omega_2\phi_{1,n} + \omega_1\phi_{2,n}, & n_1 \le n \le n_c \\ \phi_{2,n}, & n_c \le n \le n_h \end{cases} \tag{8}$$

Where n_1 and n_c are the wavenumbers within the range [n_l, n_h], with (n_1 < n_c). The weighting coefficients ω_1 and ω_2 satisfy (ω_1 + ω_2 = 1). During the transition from mismatch to matching behavior, as the wavelength gradually decreases, ω_1 decreases while ω_2 increases correspondingly. When (ω_2 = 1), the mismatch behavior disappears completely and the system fully transitions into a matched state.

3. Results and Discussion

3.1. Validity of the Numerical Simulation Method

The fracture length was set to 20.48 μm, with a horizontal lattice resolution of 1024 lu. The conversion factor between the physical system and the lattice system was 0.02 μm. The wavenumber parameters were set as (n_l = 0) and (n_h = 50). The fractal dimension was chosen as (D = 1.1), and the average aperture was 200 lu. The minimum and maximum wavenumbers in the transition region were set to (n_1 = 3) and

(n_c = 10), respectively. Based on the above parameter settings, a series of rough fractures were constructed, and the fluid flow processes were simulated using the lattice Boltzmann method.

To verify the effectiveness of the LBM in simulating nonlinear flow in rough fractures, the numerical simulation results were compared with those obtained from the Forchheimer equation. The comparison results are shown in Fig. 1. It is evident that the fitting results based on the Forchheimer equation are in good agreement with the LBM simulation results, indicating that the LBM method can effectively capture the nonlinear fluid flow behavior in rough fractures.

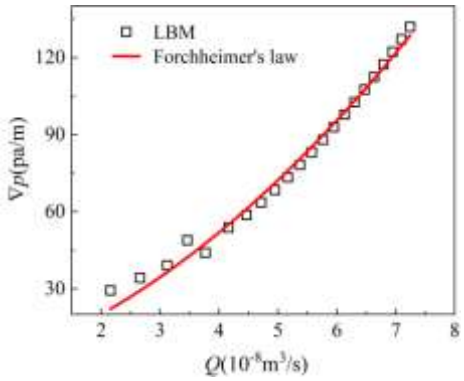


Figure 1. Comparison between LBM simulation results and Forchheimer equation fitting results

3.2. Influence of Fracture Geometric Roughness on Non-Darcy Flow Behavior

A large number of experimental studies have shown that variations in the fractal dimension of rough fractures can, to some extent, determine the degree of fracture roughness. To investigate the influence of geometric roughness on nonlinear flow behavior, five groups of rough fracture models with different fractal dimensions but identical mismatch degrees were selected for LBM seepage simulations. The minimum and maximum wavenumbers in the transition region were set to n_1=3 and n_c=10, respectively, and the fractal dimensions were set to 1.2, 1.25, 1.3, 1.35, and 1.4.

Figure 2 presents the relationship between flow rate and pressure gradient under different fractal dimensions. Specifically, Figs. 2(a) and 2(b) correspond to fracture apertures of 180 lu and 200 lu, respectively. It can be observed that, with increasing flow rate, the relationship between pressure gradient and flow rate gradually deviates from linearity, while the Forchheimer equation provides a good description of this nonlinear seepage behavior. In particular, the nonlinear coefficient B increases with increasing fractal dimension, indicating that a higher fractal dimension corresponds to a rougher fracture surface and stronger flow resistance. This enhanced resistance further intensifies inertial effects within the fracture, ultimately leading to more pronounced nonlinear flow behavior.

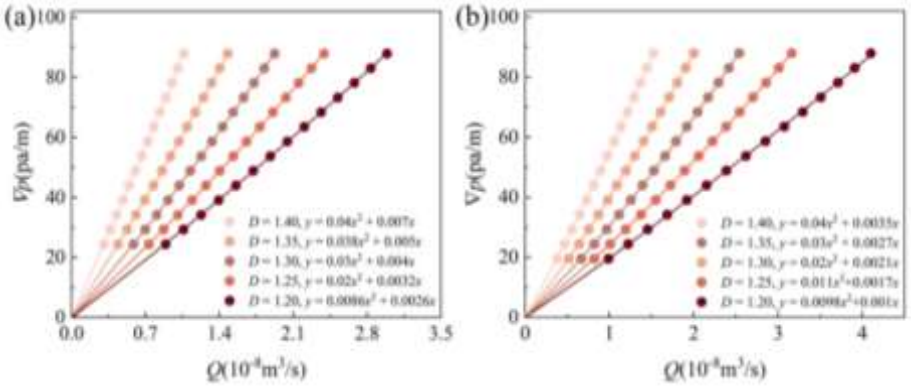


Figure 2. Relationship between flow rate and pressure gradient. (a) $\langle\delta\rangle = 180$; (b) $\langle\delta\rangle = 200$

Differences in the rough geometric morphology of fracture surfaces exert significant and complex influences on fluid seepage behavior. Therefore, to further investigate the seepage mechanisms within rough fractures, it is necessary to distinguish the individual components of roughness rather than simply attributing all effects to a generalized surface roughness parameter. As discussed previously, the influence of fracture surface roughness on fluid seepage can be decomposed into three effects: the local roughness effect, the hydraulic tortuosity effect, and the surface tortuosity effect. However, these three effects all originate from the same rough fracture geometry, and their controls on nonlinear seepage behavior are therefore inherently correlated to some extent. To further reveal the influence mechanism of fracture surface roughness on nonlinear flow, the relationships between the threefold effects and fracture surface roughness are first investigated and verified.

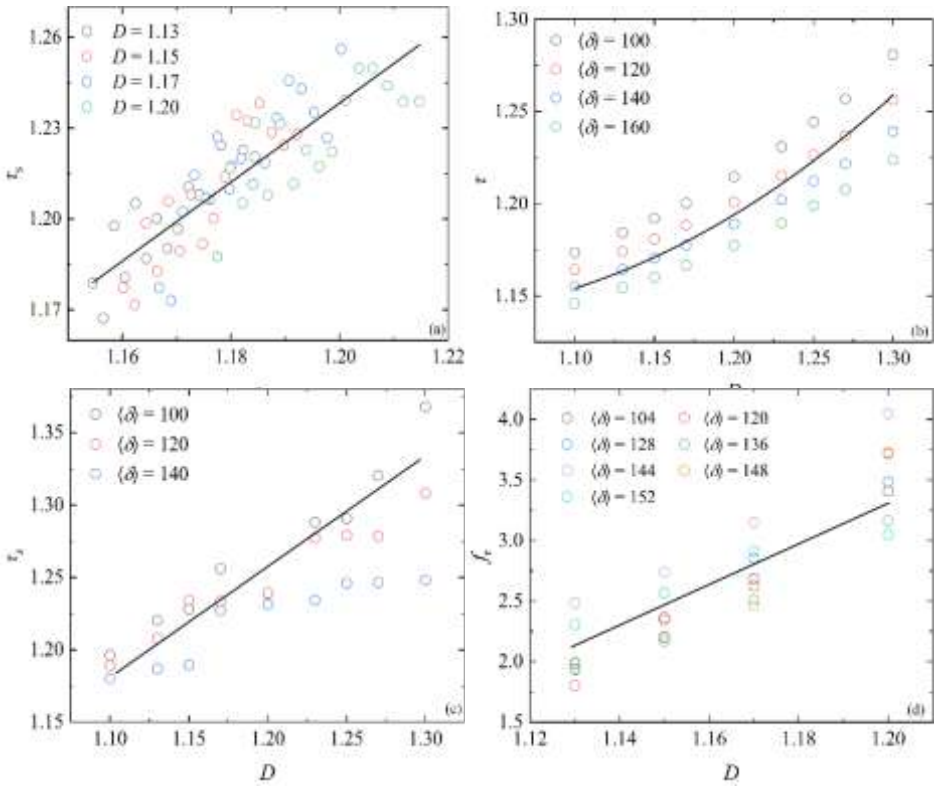


Figure 3. (a) Relationship between hydraulic tortuosity and surface tortuosity coefficient; (b) variation of hydraulic tortuosity with fractal dimension; (c) variation of surface tortuosity coefficient with fractal dimension; (d) variation of local roughness factor with fractal dimension

Figure 3 reveals the intrinsic correlation among hydraulic tortuosity, surface tortuosity coefficient, and local roughness factor. As shown in Fig. 3(a), under a constant fractal dimension, a clear positive correlation is observed between the surface tortuosity coefficient and hydraulic tortuosity. Considering the highly random nature of fracture surfaces, their geometry can be regarded as being composed of local segments distributed at different orientations around an average plane. The surface tortuosity coefficient τ_s, defined as the ratio between the actual surface area (A_s) and its projected area As on the x-y plane, serves as an important indicator of surface irregularity.

Based on this definition, an increase in hydraulic tortuosity implies that fluid streamlines within the fracture become more tortuous, leading to longer flow paths and increased geometric complexity of transport channels. This enhanced flow path complexity is associated with more pronounced surface undulations, resulting in an increase in the actual surface area, while the projected area remains nearly unchanged. Consequently, the surface tortuosity coefficient increases significantly with hydraulic tortuosity.

Figures 5(b), 5(c) and 5(d) illustrate the variation trends of hydraulic tortuosity, surface tortuosity coefficient, and local roughness factor with fractal dimension. As the fractal dimension increases, the fracture geometry becomes increasingly complex and irregular, exhibiting stronger roughness and more pronounced self-affine characteristics across multiple scales. In such high-fractal-dimension fractures, fluid migration is forced to follow more tortuous and nonlinear pathways. Streamlines no longer exhibit simple linear trajectories but instead undergo frequent deflection and flow bypassing due to combined effects of surface perturbations and local geometric constraints. This increases the overall flow path length and enhances energy dissipation, thereby increasing hydraulic tortuosity.

In addition, higher fractal dimensions introduce more complex multi-scale surface fluctuations, significantly increasing the actual surface area relative to its projected area, which leads to an increase in the surface tortuosity coefficient. Moreover, greater geometric complexity enhances microscale flow disturbances caused by irregular surface fluctuations, resulting in an increase in the local roughness factor with increasing fractal dimension.

In summary, since all three effects originate from fracture surface roughness geometry and are therefore inherently correlated in controlling nonlinear seepage behavior, one representative parameter—hydraulic tortuosity—is selected in the following analysis to further investigate the influence mechanism of fracture surface roughness on nonlinear flow behavior, thereby avoiding redundant analysis.

3.3. Influence of Hydraulic Tortuosity Effect on Fluid Flow

To investigate the influence of hydraulic tortuosity on nonlinear seepage behavior in fractures, the relationship between the non-Darcy factor E and hydraulic tortuosity τ was obtained, as shown in Fig. 4(a). The flow behavior can be divided into three typical regimes: a linear regime ($\tau < 1.06$), a linear–nonlinear transition regime ($1.06 < \tau < 1.13$), and a nonlinear regime ($\tau > 1.13$). The data points correspond to simulation results with different fractal dimensions (D = 1.1, 1.15, 1.2), reflecting the significant regulatory effect of fracture geometric complexity on nonlinear flow response.

In the linear regime, the non-Darcy factor E remains below approximately 0.1 and is nearly constant, indicating that the flow behavior within the fracture obeys Darcy law. In this stage, viscous forces dominate the system, and the migration pathway of the fluid along the main flow direction remains relatively stable. Geometric perturbations have a limited influence on the flow trajectory, and no evident nonlinear response is observed.

As the system enters the transition regime, E gradually increases with increasing hydraulic tortuosity. This trend indicates that geometric irregularities increasingly disturb the flow pathways, causing streamlines to become more tortuous. As a result, the velocity field becomes non-uniform, shear effects intensify, and localized non-Darcy behavior gradually emerges. In this stage, nonlinear mechanisms begin to contribute to the system response but do not yet dominate the overall flow behavior.

In the nonlinear regime, E exhibits a rapid increase, and the flow system shows strongly nonlinear characteristics. Streamlines deviate significantly from the main flow direction, and complex flow structures such as flow separation, recirculation, and micro-scale vortices frequently occur. In this regime, inertial effects dominate momentum transport, leading to a substantial increase in energy dissipation. Moreover, at the same level of hydraulic tortuosity, a higher fractal dimension corresponds to a larger non-Darcy factor, further confirming the strong coupling between fracture geometric complexity and nonlinear flow response.

Similarly, a higher hydraulic tortuosity leads to more complex flow pathways and stronger flow perturbations, making the transition from linear to nonlinear flow occur earlier. As a result, the critical Reynolds number decreases with increasing hydraulic tortuosity, as shown in Fig. 4(b).

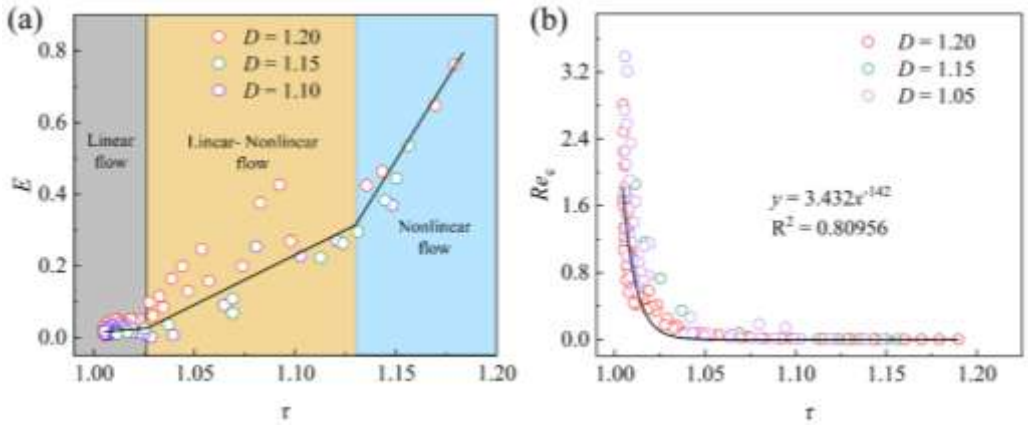


Figure 4. Variation characteristics of the non-Darcy factor E and critical Reynolds number with hydraulic tortuosity τ

3.4. Influence of Aperture Heterogeneous Distribution on Non-Darcy Flow

To further investigate the effect of aperture heterogeneity on nonlinear seepage behavior in rough fractures, LBM simulations were conducted under different degrees of aperture non-uniformity. Aperture heterogeneity directly affects the spatial distribution of local flow resistance by introducing variability in fracture opening, thereby altering flow pathways and velocity fields within the fracture.

Research has shown that, within a certain observation scale, the upper and lower surfaces of fractures exhibit mismatch characteristics, which leads to heterogeneous aperture distributions and, consequently, significantly constrains fluid transport within fracture spaces. Specifically, mismatched fracture surfaces may induce local aperture reduction or enlargement, resulting in spatially heterogeneous seepage pathways. This causes non-uniform flow velocity and distribution within the fracture, increases flow path complexity, and promotes the development of local vortices or flow bifurcation. These effects significantly enhance the nonlinear characteristics of fluid flow.

To investigate the influence of aperture heterogeneity on nonlinear seepage behavior, the fractal dimension was fixed at 1.1 and 1.2, the average aperture was set to 100 lu, and nl = 3. Multiple fracture models with different mismatch ranges were constructed by varying nc, and fluid transport processes were simulated using the LBM method. Figure 5 presents the variation of the non-Darcy factor and critical Reynolds number with aperture heterogeneity. Here, aperture roughness σ_δ is used to characterize the degree of aperture heterogeneity.

The results indicate that the spatial distribution of fracture aperture plays a decisive role in governing fluid flow behavior. When (σ_δ = 0), fracture surface roughness dominates the flow regime. As σ_δ gradually increases from zero, both surface roughness and aperture heterogeneity jointly control the flow behavior; however, the influence of aperture heterogeneity is relatively weak at the initial stage, and therefore the growth rate of the non-Darcy factor E is slow. As σ_δ further increases, numerous local constriction and dilation zones emerge within the fracture. Narrow regions increase flow resistance, while enlarged regions induce local backflow, recirculation, and vortex formation. As a result, the flow deviates increasingly from an ideal laminar state, and the controlling effect of aperture heterogeneity becomes progressively stronger.

However, when σ_δ reaches a sufficiently high level, highly permeable regions tend to become geometrically connected, forming relatively stable preferential flow channels. Consequently, the main flow paths become more concentrated, the overall flow structure stabilizes, and the enhancement of nonlinear effects gradually saturates. Therefore, the response of the non-Darcy factor E to σδ exhibits a non-monotonic "initial increase–subsequent stabilization" trend (as shown in Fig. 4(a). Similarly, as aperture heterogeneity increases, fluctuations in flow velocity become more pronounced, and local turbulence, recirculation, and flow instability are increasingly evident. This causes the flow to enter the nonlinear regime earlier, resulting in a lower critical Reynolds number. In other words, greater aperture heterogeneity facilitates the transition from linear to nonlinear flow (as shown in Fig. 5(b)).

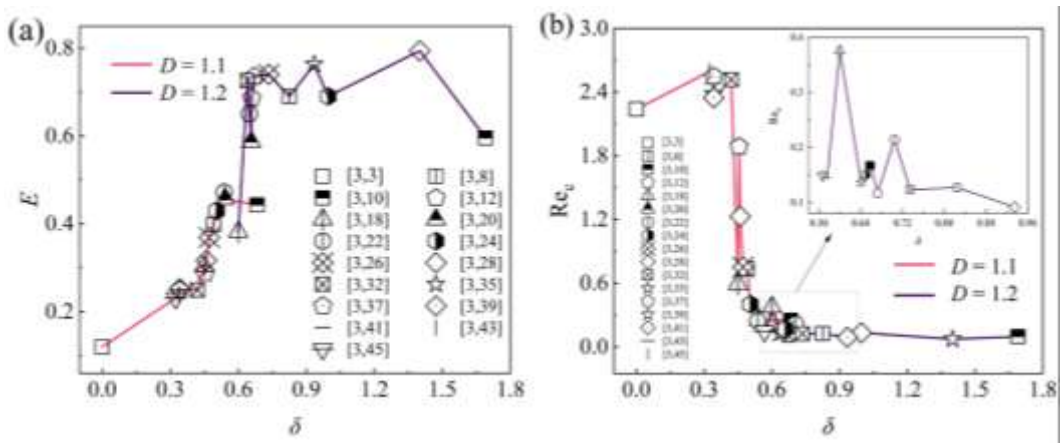


Figure 5. Variation characteristics of the non-Darcy factor and critical Reynolds number with aperture heterogeneity distribution

4. Conclusions

(1) Based on the lattice Boltzmann method (LBM), the nonlinear flow behavior in rough fractures under the combined influence of surface roughness and aperture heterogeneity was systematically investigated. The results demonstrate that the Forchheimer equation can accurately describe the nonlinear seepage characteristics in rough fractures, and its nonlinear coefficient can be effectively determined through quantitative characterization of fracture geometry.

(2) The fracture surface roughness, including local roughness, hydraulic tortuosity, and surface tortuosity, significantly enhances flow resistance and promotes inertial effects. With increasing geometric complexity, the transition from linear to nonlinear flow becomes more pronounced, and the non-Darcy effect factor E increases accordingly.

(3) Hydraulic tortuosity plays a critical role in controlling flow regime evolution by increasing flow path complexity and enhancing velocity field non-uniformity. As tortuosity increases, the critical Reynolds number decreases, indicating that the flow more readily transitions into the nonlinear regime.

(4) Aperture heterogeneity induces strong spatial variability in flow channels, leading to local acceleration, flow separation, and recirculation. The non-Darcy factor E shows a non-monotonic trend with increasing aperture heterogeneity, while the critical Reynolds number decreases overall, suggesting earlier onset of nonlinear flow.

(5) The coupling effects of fracture surface roughness and aperture heterogeneity jointly govern the evolution of flow regimes in rough fractures. This study provides a mechanistic understanding of the transition between linear and nonlinear flow, offering theoretical support for accurate evaluation of coalbed methane transport and production efficiency.

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